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PHYSICAL COMPONENTS OF THE SHEAR STRENGTH OF SATURATED CLAYS.(U)
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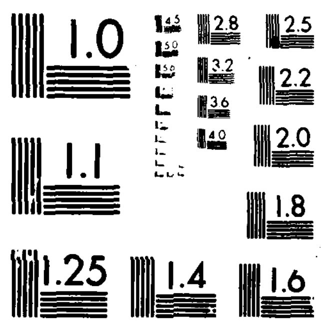

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PHYSICAL COMPONENTS OF THE SHEAR STRENGTH OF SATURATED CLAYS

MISCELLANEOUS PAPER NO. 3-428

January 1961

U. S. Army Engineer Waterways Experiment Station
CORPS OF ENGINEERS
Vicksburg, Mississippi

ERRATA IN PAPER

"PHYSICAL COMPONENTS OF THE SHEAR STRENGTH OF SATURATED CLAYS"

<u>24 April 1961</u>

Page 1, 1st paragraph, line 15	Change "souces" to "sources"
Page 1, 1st paragraph, line 19	Change "of simplifications" to "or simplifications"
Page 17, Eq. (14)	Change last right-hand member "c_e" to "c_v"
Page 52, 1st paragraph, line 5	Change "45 -1/2ϕ" to "45 - $\frac{1}{2}\phi$"
Page 75, 2d paragraph, line 3, and lines 5-6	Change "<u>strength</u> and is the peak value" to "<u>strengths</u> and are the peak values"
Page 75, 2d paragraph, line 7	Change "value of τ_f" to "values of τ_f"
Page 102, 1st paragraph, lines 6-7	Change "principal stresses" to "effective stresses"
Page 110, lines 12 and 13	Change "1/2($\sigma_1' - \sigma_3'$)" to "$\frac{1}{2}(\sigma_1' - \sigma_3')$" and "1/2($\sigma_1' + \sigma_3'$)" to "$\frac{1}{2}(\sigma_1' + \sigma_3')$"
Page 120, line 6	Change "WILSON, S. R." to "WILSON, S. D."

(6)

PHYSICAL COMPONENTS OF THE SHEAR STRENGTH OF SATURATED CLAYS.

(10) M. Juul / Hvorslev

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U. S. Army Engineer Waterways Experiment Station

CORPS OF ENGINEERS

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PREFACE

The preparation of this paper was assigned and authorized by the Chief of Engineers, U. S. Army, as a part of the contribution by the Corps of Engineers, Department of the Army, to the ASCE Research Conference on Shear Strength of Cohesive Soils, held at the University of Colorado, Boulder, Colorado, in June 1960. The paper, which will be published in the Proceedings of the conference, deals with the shear strength of saturated, remolded, and reconsolidated clays, and with related properties which influence the shear strength. It is in part a restatement of the results of earlier investigations by the author and in part a review of recent research by others. The review was performed and the paper was prepared at the U. S. Army Engineer Waterways Experiment Station under the general direction of Colonel Edmund H. Lang, Director, Mr. J. B. Tiffany, Technical Director, and Mr. W. J. Turnbull, Chief of the Soils Division. The assistance of the Reproduction and Reports Branch is gratefully acknowledged, and special recognition is due Messrs. R. A. Daumer and R. J. Alvarado for tracing the figures and Miss Katharine H. Jones for editing the paper.

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PHYSICAL COMPONENTS OF THE SHEAR STRENGTH OF SATURATED CLAYS

by

M. Juul Hvorslev,* F. ASCE

INTRODUCTION

During the period 1933-1936 the writer was engaged in research on the physical properties of clays at the Technical University of Vienna, Austria, under the general direction of Professor K. Terzaghi. The results of this research were published as a thesis in 1937, and abstracts thereof were published as conference papers in 1936 and 1938. The writer has had but little opportunity personally to perform additional experimental investigations of the shear strength of clays, and this paper is to a large extent a re-evaluation and restatement of some of the results of the above-mentioned research. The paper deals primarily with the physical components of the shear strength of remolded, saturated clays and with the various factors which influence these components. The results of subsequent research by others are taken into consideration and summarized when appropriate, but the paper is not a complete review of the very extensive and important research on shear strength of saturated clays performed during the last twenty years. The souces of error in the tests performed by the writer are discussed, and emphasis is placed on clarification of the assumptions and limitations relating to the conclusions and formerly proposed criteria for the shear strength of saturated clays. New data and illustrations are included, but many figures are copies of simplifications of the original figures in the 1937 publication.

Most of the data presented in this paper concern the properties of remolded clays. As demonstrated by A. CASAGRANDE (1932), the results of tests on remolded clays cannot be used directly for solution of practical problems involving undisturbed clays. Nevertheless, many relations concerning physical properties of clays were first determined by means of tests on remolded clays, and they apply with minor modifications and limitations to undisturbed clays, but the coefficients entering these relations may be quite different for remolded and undisturbed clays. The use of remolded soils in basic research

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has important advantages, especially in regard to uniformity of the test specimens, control of their stress history, and separation of the influence of the many variables which govern the deformation and strength characteristics of soils. However, it is essential that research on remolded soils be supplemented by research on undisturbed soils, and that the results of the latter be verified and amended by observations and analyses of the behavior of prototype structures.

1. INTRINSIC FORCES AND STRUCTURE OF CLAYS

Intrinsic Forces

Clarification of the physicochemical constitution of clays and of the intrinsic forces acting in the soil-water system is of utmost importance to a better understanding of the deformation and strength characteristics of these soils. Earlier investigations of this problem by GOLDSCHMIDT (1926), TERZAGHI (1925, 1931, 1941), ENDELL (1936), FREUNDLISH (1935), and others have in recent years been supplemented by extensive research in the fields of colloid chemistry, mineralogy, soil physics, and soil mechanics. Significant results have been obtained, and among the many papers dealing in part or whole with the subject, reference may be made to those by MASON and WARD (1954), ROSENQUIST (1955), TAN (1957), LAMBE (1958), and to those in a symposium on physicochemical properties of soils published in the Journal of the Soil Mechanics and Foundations Division, ASCE, April 1959. It is difficult to summarize the results of this research because of the remaining uncertainties and differences in interpretation of the observed phenomena.

The conception that the usually flaky particles of the clay minerals are surrounded by double layers of bound and partially bound water has been strengthened and further detailed. It is agreed that the clay particles are covered with a thin film of strongly bound or adsorbed water which in turn is surrounded by a thicker film of partially bound water. The structure of the water in these layers is different from that of free liquid water; the adsorbed water has some properties similar to those of ice, but opinions vary in regard to its actual structure, density, and viscosity. However, it has been suggested that the adsorbed water is so strongly bound to the clay particles that it is moved by diffusion rather than viscous flow, and that it prevents actual contact between the clay particles at pressures normally encountered in soil deposits; BOLT (1956), ROSENQUIST (1959). The water in the adsorbed and partially bound layers primarily serves to transmit and modify the electrochemical forces between the clay crystals, and the bipolar character of the water molecules and the ions in the water are of primary importance to this function. The principal intrinsic forces are the van der Waals forces and the Coulombic forces. The van der Waals forces normally cause attraction but decrease extremely rapidly with distance, and they predominate at points of very small spacing of the particles. The Coulombic forces are (1) electrostatic attraction between the positive charges at the edges of one particle and the

negative charges at the flats or surfaces of other particles, and (2) electrostatic repulsion between two surfaces or between two edges of adjacent particles. Additional cohesive forces may in some cases be caused by hydrogen or potassium bonds or may result from cementation by organic or inorganic compounds.

The magnitude of the resultant intrinsic forces depends on the type of clay minerals, the size and corresponding specific surface of the particles, the type of ions adsorbed on the surfaces of the clay crystals, the type and concentration of the ions in the water, and on temperature. Since the Coulombic forces decrease with the square of the distance and the van der Waals forces with about the sixth power of the distance from the particles, the intrinsic forces are governed by the points of shortest distance between the particles rather than by their average spacing; that is, the intrinsic forces depend not only on the void ratio but also on the geometric arrangement of the particles or the soil structure; see LAMBE (1953, 1958), ROSENQUIST (1959). External and gravity forces influence the intrinsic forces when the external forces cause a change in the effective stresses in the soil-water system and a corresponding change in the spacing of the clay particles.

Considering the deformation characteristics of clays, it has been proposed by GOLDSTEIN (1957) and others that the intrinsic forces be divided into two groups, one producing elastic bonds and the other forming viscous bonds, but it has not yet been possible to identify the basic forces and conditions which cause formation of the two types of bonds. NASCIMENTO (1953) and BOROWICKA (1959) have suggested that changes in free energy and corresponding forces at the interface of water and soil particles can produce tension in the bound water and a corresponding cohesion at very close spacings of the particles. However, it has not yet been verified experimentally that such a tension in the bound water exists, and the hypothesis has not been correlated with other observed phenomena or with the concepts summarized in the foregoing paragraphs.

The intrinsic forces in the soil-water system may in some cases undergo thixotropic changes, defined by FREUNDLICH (1935) as isothermal, reversible sol-gel transformations; that is, the strength or coherence of a clay may be decreased, without any change in water content or temperature, by large and rapid deformations, but the strength is gradually regained when the deformations cease or the rate of deformation decreases. The phenomenon may be visualized as a disturbance and subsequent re-establishment not only of the

arrangement of the clay particles but also of the structure of the bound water with consequent changes in the transmission of electrochemical forces between the particles. The thixotropic properties of clays depend primarily on the type and concentration of ions in the pore water and adsorbed on the surface of the clay minerals. Recent data and concepts relating to thixotropic changes in strength of soils are presented in the papers by SEED and CHAN (1959-A) and MITCHELL (1960). The latter has demonstrated that the pore-water pressure increases during a thixotropic disturbance of a cohesive soil, and that this pressure decreases during a subsequent thixotropic hardening of the soil, which in a physical sense explains the changes in strength. Similar observations have also been made by BISHOP, ALPAN, BLIGHT, and DONALD (1960).

Clay Structure

TERZAGHI (1925) suggested various single-grain and honeycomb structures* as an explanation of the great variations in void ratio and behavior of soils. Goldschmidt maintained in published lectures that the flaky clay particles in sensitive clays lean upon each other and form an open and unstable structure; see ROSENQUIST (1959). CASAGRANDE (1932) proposed a compound honeycomb structure of silt grains and flocculated clay particles for sensitive undisturbed clays.

The structure of a clay is disturbed by remolding, and it may be assumed that the clay particles have no preferred orientation after a thorough remolding, considering the entire remolded mass. However, a definite orientation may be imparted to the clay particles during subsequent consolidation and strength tests on the remolded clay. This possibility was investigated by the writer by means of slaking tests. A silty clay was remolded at a water content close to that of the liquid limit and then reconsolidated by confined compression. A thin slice was cut from the test specimen in a direction parallel to that of the principal consolidating stress, and another slice was cut perpendicular thereto. These slices were partially air-dried and then laid in water and allowed to slake. As seen in Figs. 1-A and 1-B, the expansion occurs in a direction parallel to the principal consolidating stress, and the fissures are perpendicular thereto. A ball was formed of another part of the

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* The term "structure" refers here to the geometric structure or arrangement of the soil particles, which also is called "fabric" in some recent publications, where "structure" includes both the geometric structure and the force structure; see LAMBE (1958).

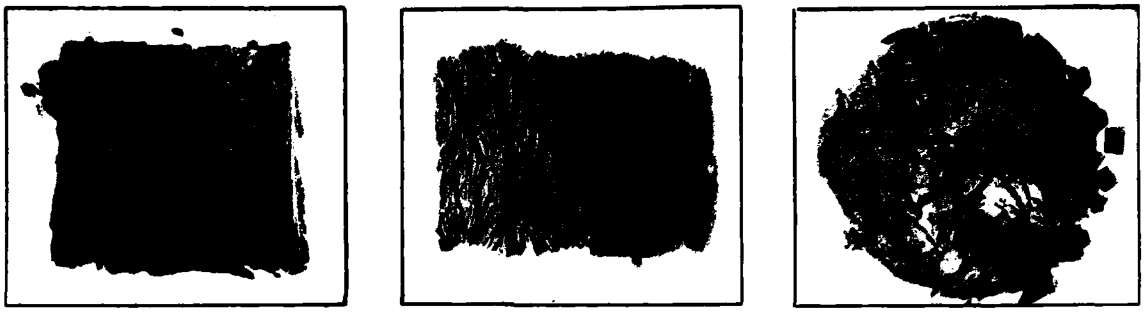
remolded clay and air-dried. In this case the consolidation stress, i.e. capillary pressure, is of equal magnitude in all directions, and a slaking test on a thin slice cut from the ball shows no preferred orientation of deformations and fissures, Fig. 1-C. As shown in Fig. 2, similar results were also obtained in slaking tests on partially dried slices of undisturbed clays.

The conclusions drawn from these experiments were that the clay particles in the remolded and uniaxially reconsolidated clay have a preferred orientation perpendicular to the direction of the principal consolidating stress, and that the consolidated test specimen may have anisotropic strength, deformation, and permeability characteristics. These conclusions were substantiated by the results of unconfined compression tests described in Section 6.

It is probable that shear strains promote orientation of the clay particles in a direction parallel to that of the principal strain, provided the strains are active over a sufficiently long period of time. If the strains also are very large, they may cause formation of slickensided failure surfaces, as found in torsion shear tests at the USAE Waterways Experiment Station, Fig. 3; HVORSLEV and KAUFMAN (1952). However, slickensided failure surfaces were not formed when the normal stress on the surface was less than 0.5 kg/cm^2.

Recent research has yielded very interesting hypotheses and data on the structure of both undisturbed and compacted clays and on the influence of the structure on the deformation and strength characteristics of clays. LAMBE (1953, 1958) has suggested that clays which were flocculated during sedimentation or compacted at water contents less than optimum water content have a random orientation of the clay particles or a "cardhouse" type of structure. Conversely, clays which were dispersed during sedimentation or compacted at a water content greater than the optimum water content have a parallel orientation of the particles. Possible variations from these general rules are discussed in a paper by SEED and CHAN (1959-B), who also found that although the structure may have a pronounced influence on the deformation characteristics of compacted clays, it has relatively little influence on the maximum strength because a reorientation of the particles in the failure zone may take place during a shear or triaxial test.

ROSENQUIST (1959) has developed ingenious techniques for obtaining stereophotographs of the structure of undisturbed clays by means of an electron microscope. He found that the structure of marine clays in general is a random orientation of the particles and resembles the cardhouse structure



A
PRESSURE PERPENDICULAR TO PLANE OF PICTURE

B
PRESSURE HORIZONTAL IN PLANE OF PICTURE

C
PRESSURE UNIFORM IN ALL DIRECTIONS

SLAKING TESTS ON REMOLDED AND RECONSOLIDATED VIENNA CLAY

FIG. 1

A
SILTY CALCAREOUS CLAY

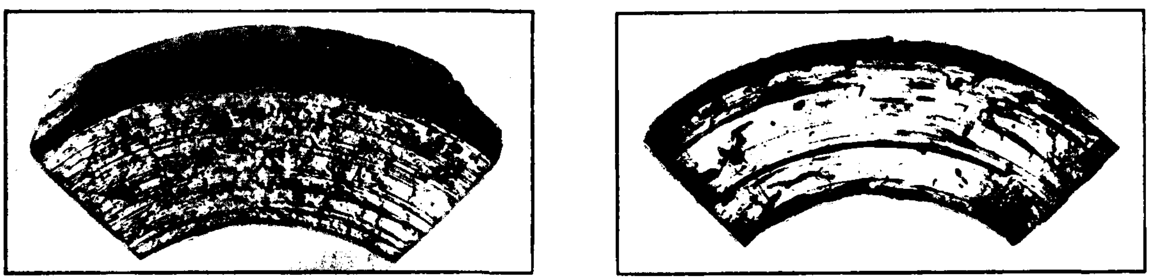
B
VIENNA SILTY CLAY

C
CRUMBLY CALCAREOUS CLAY

HORIZONTAL SLICES IN LEFT HALF AND VERTICAL SLICES IN RIGHT HALF OF PHOTOS

SLAKING TESTS ON PARTIALLY DRIED SLICES OF UNDISTURBED CLAYS

FIG. 2



A
LAKE PROVIDENCE CLAY
VERTICAL LOAD 2.0 KG/CM^2

B
ATLANTIC MUCK, PANAMA CANAL
VERTICAL LOAD 5.0 KG/CM^2

FORMATION OF SLICKENSIDED FAILURE SURFACES IN TORSION SHEAR TESTS

FIG. 3

suggested by GOLDSCHMIDT, LAMBE (1958), and TAN (1957). However, the photographs also show that groups of particles may form "packets" with a parallel alignment of the flaky clay particles. BUESSEM and NAGY (1953), SALAS and SERRATOSA (1953), and MITCHELL (1956) have used X-ray diffraction or polarized light and the birefringent properties of clay crystals to investigate the relative orientation of clay particles. They found that the particles in remolded and uniaxially reconsolidated clays are oriented perpendicularly to the direction of the principal consolidating stress. MITCHELL (1956) also investigated the structure of many undisturbed clays. He found that some clays have a completely random orientation of the particles and that others have various degrees of a preferred orientation of the particles. The degree of parallel orientation of the particles is uniform in some clays; in others, this degree of orientation varies from spot to spot or zone to zone.

2. DEFINITIONS AND CONCEPTS RELATING TO SHEAR STRENGTH

Shear Strength and Effective Stresses

The failure conditions for a soil may be expressed in terms of a limiting shear stress, called shear strength, or as a function of the principal stresses. The two forms of the failure conditions are often but not always interchangeable. The shear strength of a soil, τ_f, may be defined as the shear stress in the plane of failure at the time of failure. In case of direct or torsion shear tests, the plane between the stationary and moving parts of the equipment is considered as the enforced plane of failure. The stress-displacement curve for this plane usually has a well-defined peak point, Fig. 4, and the corresponding maximum value of the shear stress is considered to be the shear strength for the purpose of the investigations discussed in this paper. It should be noted that the shape of the stress-strain curve depends on the type of loading, rate of deformation, and drainage conditions. The oldest and still widely used expression for the shear strength is the Coulomb failure criterion,

$$\tau_f = c + \sigma_f \tan \phi \tag{1}$$

where c is the cohesion, σ_f the normal stress on the failure surface, and ϕ the angle of internal friction. This equation is relatively simple, but the values of c and ϕ depend on many factors and may vary between wide limits; also the field of application of Eq. 1 is limited to conditions duplicating those existing during the test in which the values of the coefficients were determined.

A failure criterion of greater general applicability is obtained by expressing the shear strength as a function of the effective normal stress, σ_f', in accordance with Terzaghi's fundamental concept that the strength and deformation characteristics of soils are governed by the effective stresses rather than the total stresses. This paper deals solely with saturated soils for which

$$\sigma_f' = \sigma_f - u \tag{2}$$

and

$$\tau_f = c' + \sigma_f' \tan \phi' \tag{3}$$

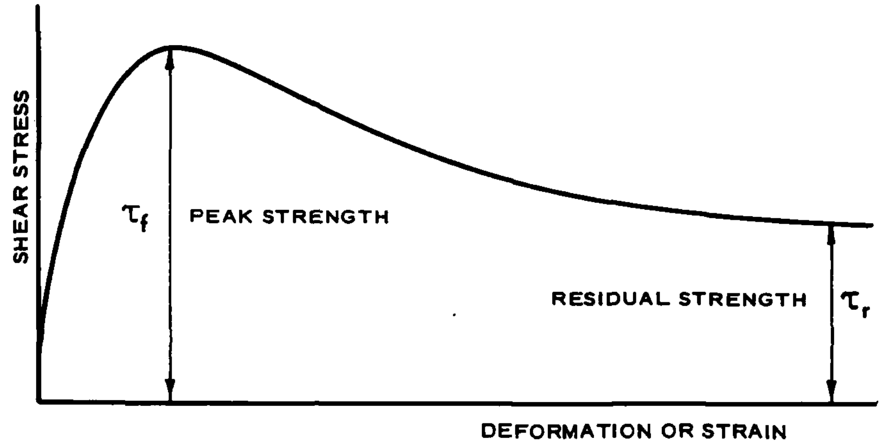


FIG. 4. SHEAR STRESS – DEFORMATION DIAGRAM

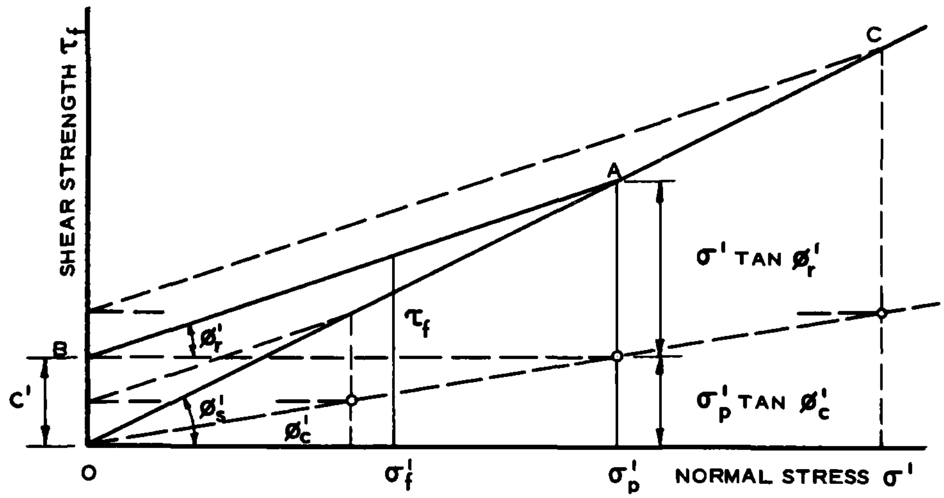


FIG. 5. COULOMB SHEAR STRENGTH DIAGRAM

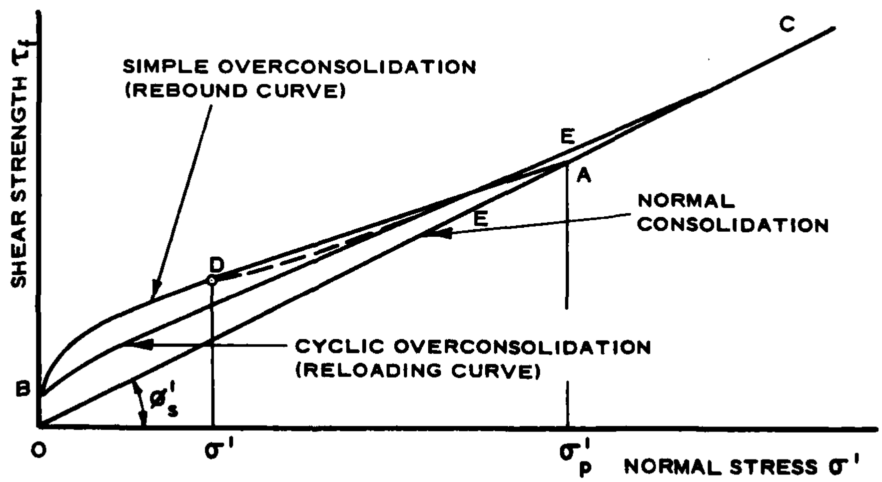


FIG. 6. SHEAR STRENGTH HYSTERESIS LOOP

where u is the pore-water pressure, which is assumed to be zero in the case of fully drained tests. The pore-water pressures thus defined or measured probably represent the pressures in the free or lightly bound pore water; the pressures in the more strongly bound water at points of minimum distance between the soil particles are not yet definitely known. The effective stress concept may be said to be semiempirical, but its practical validity has been demonstrated in many laboratory tests and field observations.

Influence of the State of Consolidation

A graphical representation of Eq. 3 is shown in Fig. 5. The line OAC is the shear strength line for a normally consolidated clay, for which $\phi' = \phi'_s$ and $c' = 0$. The line BA is the shear strength line for the same clay preconsolidated at the pressure σ'_p and having $\phi = \phi'_r$. If the clay is preconsolidated at other pressures, shear strength lines parallel to BA are obtained, as indicated by the dashed lines in the figure. This means that c' is proportional to σ'_p or

$$c' = \sigma'_p \tan \phi'_c \tag{4}$$

and Eq. 3 may then be written

$$\tau_f = \sigma'_p \tan \phi'_c + \sigma'_f \tan \phi'_r \tag{5-A}$$

For a normally consolidated clay $\sigma'_p = \sigma'_f$ and

$$\tau_f = \sigma'_f \tan \phi'_c + \sigma'_f \tan \phi'_r \tag{5-B}$$

The first term on the right side of Eq. 5-B may be considered as the cohesion component and the second term as the friction component, and the anomaly of having two angles of internal friction, ϕ'_s and ϕ'_r, for the same clay and no cohesion for a normally consolidated clay is thereby eliminated. As shown later, the shear strength lines BA in Fig. 5 are simplifications of actual test results, and they represent changes in both the friction and cohesion components. Therefore, the values of c' and ϕ'_r are not the actual cohesion or angle of internal friction but merely mathematical components; and c' is now called the cohesion intercept and the angles of inclination of the shear strength lines are called the angles of shear strength. The form and

general concept of Eq. 5-A were first proposed by Tiedemann and were based on extensions of investigations by Krey; see SEIFERT (1933, p. 41) and BRENNECKE-LOHMEYER (1938).

The shear strength lines shown in Fig. 4 and Eq. 5 are based on the results of fully drained tests. OHDE (1955) emphasizes that Tiedemann performed fully drained tests on normally consolidated test specimens, but started tests on overconsolidated test specimens immediately after reducing the total normal stress from σ_p to σ_f and attempted to maintain a constant water content equal to that existing at the preconsolidation pressure. This is correct; but if the water content actually were held constant, the values of σ_f' and τ_f would also be practically constant and equal to those existing at the preconsolidation pressure; that is, the shear strength lines BA would be nearly horizontal, as found by the writer and others. Tiedemann obtained appreciable slopes of the lines BA, which indicate that partial consolidation or swelling and equalization of the pore-water pressures had taken place during the test. The writer therefore took the liberty of calling Eq. 5 the Krey-Tiedemann failure criterion.

The shear strength relations shown in Fig. 5 and in Eqs. 3 and 5 are simple and practical, but they furnish only approximate values of the shear strength of overconsolidated soils. As first shown by TERZAGHI (1929), the shear strength diagram for a series of test specimens overconsolidated to correspond to various points on the rebound and reloading branches of the pressure-void ratio diagram, Fig. 10, forms a hysteresis loop similar to that shown in Fig. 6. This diagram shows that different values of τ_f can be obtained for identical values of σ_p' and σ_f', and that the actual shear strength lines may deviate appreciably from the straight lines shown in Fig. 5 when σ_f' is small compared to σ_p'. The shear strength lines of the hysteresis loop have much greater curvature when the shear strengths are determined by undrained tests and plotted against total normal stresses, σ_f ; see Fig. 31.

Shear strength lines with a slight double curvature are often obtained in tests on undisturbed test specimens, which may be explained in the following manner. Assume that the clay is preconsolidated at the vertical pressure σ_p' and that the current overburden pressure for the test specimens is σ_d', Fig. 6. The shear strength line DB on the rebound branch is obtained when tests are performed for values of σ_f' smaller than σ_d', but the shear strength line will pass from D to E on the reloading branch on the hysteresis loop when tests are performed for values of σ_f' greater than σ_d'.

Triaxial Stresses and Strengths

The foregoing discussion applies primarily to the results of direct box or torsion shear tests. The results of the now commonly used triaxial tests are usually evaluated by means of the Mohr diagram, as shown in Fig. 7 for an overconsolidated soil and simplified conditions corresponding to those in Fig. 5. If the angle α between the direction of the major principal stress and the planes of failure is measured or assumed, the shear stress and corresponding normal stress on a failure plane can then be determined graphically as shown in Fig. 7 or by

$$\tau_f = \frac{1}{2}(\sigma_1' - \sigma_3') \sin 2\alpha \tag{6}$$

$$\sigma_f' = \frac{1}{2}(\sigma_1' + \sigma_3') - \frac{1}{2}(\sigma_1' - \sigma_3') \cos 2\alpha \tag{7}$$

The shear strength can then be determined by a diagram similar to that in Fig. 4, but it is noted that the maximum values of τ_f and $(\sigma_1' - \sigma_3')$ occur at the same time or strain when α is a constant throughout the test. In case the soil has isotropic strength properties which conform to the Coulomb failure condition, Eq. 3, it can be shown mathematically that the optimum value of α is given by

$$\alpha = 45 - \frac{1}{2}\phi' \qquad \text{or} \qquad \phi' = 90 - 2\alpha \tag{8}$$

These equations can theoretically be used for computing α when ϕ' is known, or vice versa. However, Eq. 8 is not correct when the soil has anisotropic strength properties or when nonuniformities in stress distribution or properties exist in the test specimen; see Section 6.

The plane corresponding to the point of tangency of the Mohr envelope has the inclination $\alpha_m = (45 - \frac{1}{2}\phi_m')$, where ϕ_m' is the angle of inclination of the Mohr envelope, but this does not signify that the Mohr envelope is the shear strength line and that ϕ_m' is the angle of internal friction unless the inclination of the failure planes also is α_m and the soil has isotropic strength properties. As seen in Fig. 6, the shear strength line intersects the Mohr circles when α is either smaller or larger than α_m . Some investigators use the shear strength angle for normal consolidation, ϕ_s' , and others use ϕ_r' or ϕ_m' and corresponding values of α for evaluation of triaxial test results. According to the hypothesis proposed by the writer and

described later, the effective angle of internal friction, ϕ'_e , is smaller than ϕ'_r and ϕ'_m ; and it has also been found that the measured values of α for remolded clays usually are larger than α_m , although they are subject to considerable variation. The difference between assumed and actual values of α may result in appreciable variations in the computed values of τ_f and σ'_f , but it should be noted that these variations cause only a slight shift in the position of the shear strength line. Although the actual shear strength line may be located slightly below the Mohr envelope, the latter is usually assumed to be the shear strength line in evaluation of triaxial test results for practical purposes.

Difficulties connected with determination of correct values of τ_f and σ'_f from results of triaxial tests can be avoided by expressing the failure conditions in terms of the principal stresses. When the Mohr envelope is a straight line with the inclination ϕ'_m and the cohesion intercept c'_m , the failure condition in terms of principal stresses is

$$(\sigma_1 - \sigma_3) = (\sigma'_1 - \sigma'_3) = 2c'_m \cos \phi'_m + (\sigma'_1 + \sigma'_3) \sin \phi'_m \qquad \text{(9-A)}$$

The actual Mohr envelopes are often slightly curved, and their replacement with straight lines is equivalent to assuming that the soil conforms to the Coulomb failure criterion. Therefore, Eq. 9-A is called the Mohr-Coulomb failure condition. This equation may also be written in the following form

$$\frac{1}{2}(\sigma'_1 - \sigma'_3) = c_s + \frac{1}{2}(\sigma'_1 + \sigma'_3) \tan \beta_s \qquad \text{(9-B)}$$

By plotting $\frac{1}{2}(\sigma'_1 - \sigma'_3)$ versus $\frac{1}{2}(\sigma'_1 + \sigma'_3)$, as shown in Fig. 34, a straight line is obtained which has the angle of inclination β_s and the intercept c_s . The parameters ϕ'_m and c'_m can then be determined by comparison of Eqs. 9-A and 9-B.

$$\sin \phi'_m = \tan \beta_s \qquad \text{and} \qquad c'_m = c_s / \cos \phi'_m \qquad \text{(9-C)}$$

Such a diagram is much easier to draw than a series of Mohr circles, and it facilitates averaging of scattered test results and determination of the mean values of the parameters. Examples of application of this method for evaluation of the results of triaxial strength tests are described by KYVELLOS (1956), who indicates that the method originally was suggested by A. Caquot.

A similar method was developed by SKEMPTON and BISHOP (1954), who transformed Eq. 9-A into

$$\frac{1}{2}(\sigma_1' - \sigma_3')(1 - \sin \phi_m') = c_m' \cos \phi_m' + \sigma_3' \sin \phi_m' \tag{9-D}$$

A plot $\frac{1}{2}(\sigma_1' - \sigma_3')$ versus σ_3' yields a straight line, from the inclination and intercept of which the values of ϕ_m' and c_m' can be determined, as illustrated in Fig. 27. Finally, Eq. 9-A may be written

$$\sigma_1'(1 - \sin \phi_m') = 2c_m' \cos \phi_m' + \sigma_3'(1 + \sin \phi_m') \tag{9-E}$$

and plots of σ_1' versus σ_3' form straight lines as shown in the Rendulic diagram, Fig. 38. Further details of evaluation of the results of triaxial strength tests are discussed in Section 9.

Conversely, the results of direct and torsion shear tests cannot be expressed in terms of principal stresses without knowing or assuming the angle of inclination of the failure planes and/or the effective angle of internal friction. For the conditions and assumptions shown in Fig. 8, where ϕ_r' is the angle of inclination of the shear strength line and ϕ_e' is the effective angle of internal friction, the failure criterion in terms of principal stresses is

$$(\sigma_1' - \sigma_3') = 2c_r' \frac{\cos \phi_r'}{\cos (\phi_r' - \phi_e')} + (\sigma_1' + \sigma_3') \frac{\sin \phi_r'}{\cos (\phi_r' - \phi_e')} \tag{10}$$

Changes in α_e and ϕ_e' may cause appreciable changes in $(\sigma_1' + \sigma_3')$ and the Mohr circles, but only relatively small changes in the position and inclination of the Mohr envelope.

Volume Changes and Surface Energy

Volume changes. Changes in pore-water pressure and/or volume of the soil during shear tests constitute important factors in evaluation of the test results. These changes are summarized below in a qualitative sense in order to facilitate further definitions and explanations of observed phenomena.

The pore-water pressure increases and/or the void ratio decreases when the average external pressure is increased, and vice versa.

The pore-water pressure increases and/or the void ratio decreases when

normally consolidated or slightly overconsolidated clays are subjected to an increase in shearing stresses.

The pore-water pressure decreases and/or the void ratio increases when strongly overconsolidated clays are subjected to an increase in shearing stresses.

A stress reversal or decrease in shearing stresses may produce changes in pore-water pressures and void ratios which are of the same sign but of much smaller magnitude than those caused by a comparable increase in shearing stresses.

The pore-water pressure increases with increasing time when normally consolidated or slightly overconsolidated clays are subjected to constant shearing stresses and drainage is prevented.

It is not yet known whether the pore-water pressure increases or decreases with increasing time when strongly overconsolidated clays are subjected to constant shearing stresses and drainage is prevented.

Surface energy. A change in the volume of a test specimen signifies addition or expenditure of surface energy. TAYLOR (1948) demonstrated that consideration of volume changes and corresponding changes in surface energy during shear tests on sands can explain the difference in shear strength of dense and loose sand. This concept was later formulated by BISHOP (1950), who introduced a strength component, called the dilatation component or the surface energy correction* which is denoted by τ_d in this paper. Assume that the change in thickness of a direct shear test specimen is dy during a time interval dt at failure, and that the lateral displacement in shear is dx during the same time interval, then the surface energy is $(A \cdot \sigma_f' \cdot dy)$ and the internal energy corresponding to τ_d is $(A \cdot \tau_d \cdot dx)$, where A is the area of the test specimen. If it is assumed that the two energy quantities are equal, then τ_d is defined by

$$\tau_d = \sigma_f' \frac{dy}{dx} \tag{11}$$

BISHOP and ELDIN (1953), and BISHOP (1954) have shown that with similar

* Objections have been raised to the term "dilatation" because soils may undergo either a decrease or an increase in volume during strength tests. The term "surface energy correction" refers to a correction of the friction and cohesion components, but it is a component and not a correction of the total shear strength. Misunderstandings may be avoided by using the term "volume change component."

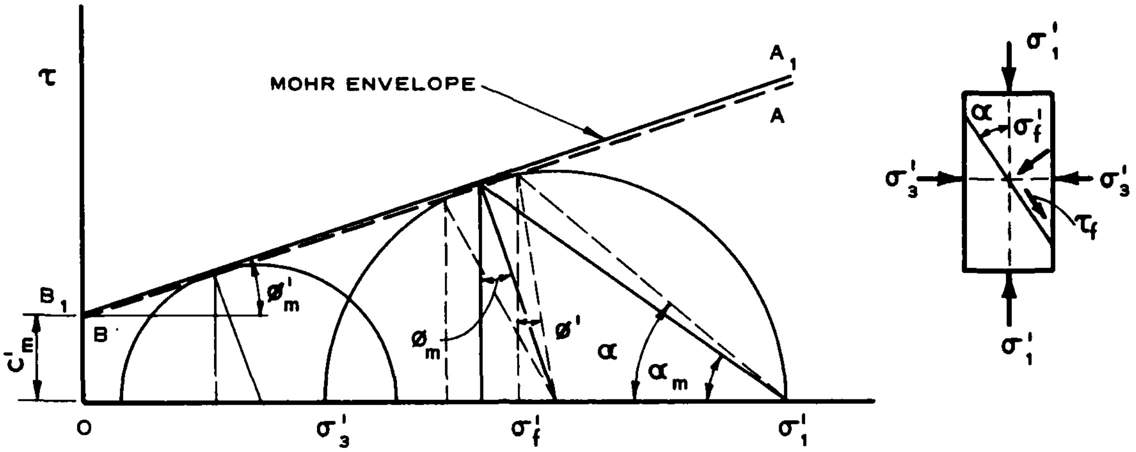


FIG. 7. MOHR DIAGRAM FOR TRIAXIAL TESTS

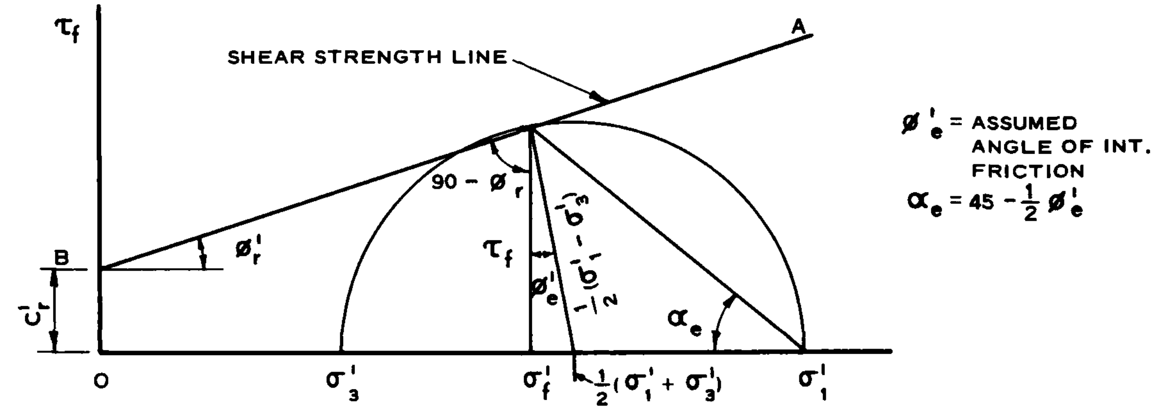


FIG. 8. MOHR DIAGRAM FOR DIRECT SHEAR TESTS

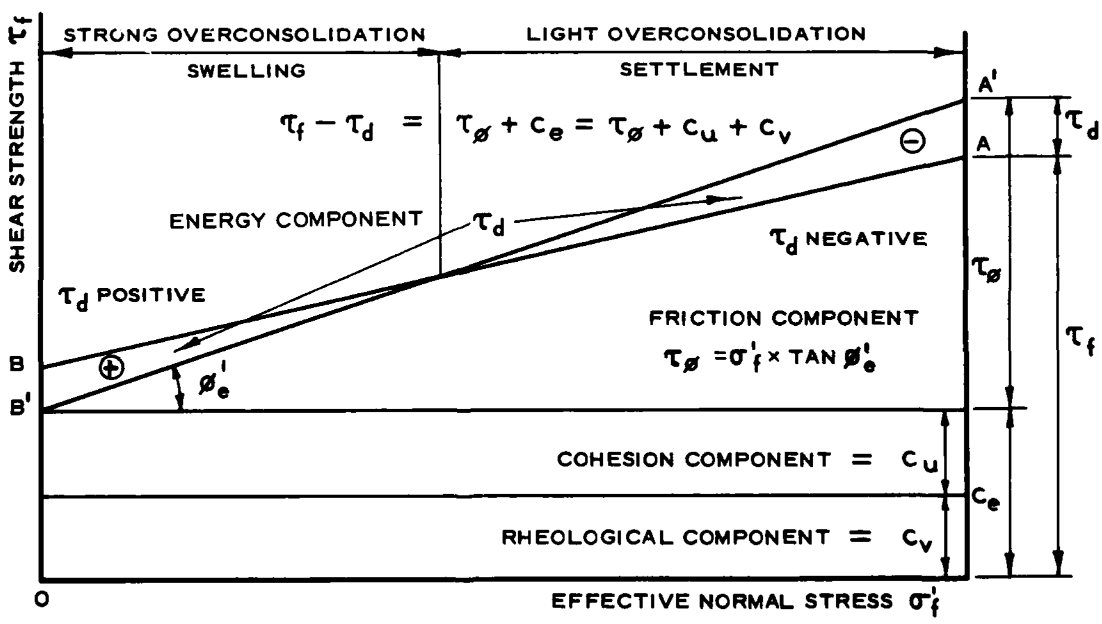


FIG. 9. COMPONENTS OF SHEAR STRENGTH AT CONSTANT VOID RATIO

assumptions the surface energy correction for triaxial tests can be expressed by

$$\tau_d = \sigma_3' \frac{dv}{d\epsilon} \tag{12}$$

where dv is the rate of volume strain and $d\epsilon$ the rate of axial strain at failure. Surface energy corrections are also made at stresses and strains before and after failure in order to obtain stress-strain curves for the individual strength components. The component, τ_d , is positive for a volume increase, negative for a volume decrease, and zero for constant volume or undrained tests.

Introduction of the surface energy component in the evaluation of shear tests on sands furnishes an explanation of the difference in strength of dense and loose sands. The resulting angle of internal friction corresponds to the ultimate or residual shear strength of dense sands, τ_r in Fig. 4; see also BISHOP (1950). The peak shear strength and the residual shear strength are identical in the case of loose sands. It is to be noted that both friction and surface energy components of sand are linear functions of the effective stresses, and it is a matter of definition or choice whether in practical applications the strengths should be expressed by a friction component based on the peak strength or on a surface energy component plus a friction component based on the residual strength.

GIBSON (1953) and others have extended the concept of a dilatation component or surface energy correction to the evaluation of the results of drained shear tests on clays. The writer has previously suggested, HVORSLEV (1953), that a part of the volume change of clays during shear is an indirect consequence of the shearing strains, and that a corresponding part of the surface energy is absorbed or contributed by internal forces as in consolidation tests and should be disregarded in computing the surface energy component. It is possible that a part of the volume change of clays during shear is caused by interference or interlocking, as in sands, but it is also probable that the shearing strains cause a partial disturbance and thixotropic weakening of the bonds or a decrease in the effective stresses between the clay particles. It is known from consolidation tests that such a disturbance promotes additional consolidation of normally consolidated or slightly overconsolidated soils and additional swelling of strongly overconsolidated clays.

The external energy supplied during a volume decrease of the test specimen may, at least in part, be absorbed internally by consolidation and produce

an increase in cohesion. The external energy expended during a volume increase may in part be supplied by internal swelling pressures and produce a decrease of the cohesion. On the other hand, introduction of the dilatation component or surface energy correction defined by Eqs. 11 and 12 primarily changes the angle of shear strength; see Fig. 9.

In the evaluation of slow, drained shear tests it is usually assumed that the excess pore-water pressure at failure is zero. However, measurable rates of volume change at failure signify the existence of remanent, positive or negative, excess pore-water pressures. These pore-water pressures are a function of the rates of volume change and the properties of the soil, and theoretically they can be computed, but it is difficult to do so when the volume changes are not uniformly distributed throughout the test specimen. Consideration of the remanent excess pore-water pressures would increase the angle of inclination of the shear strength line, and it is probable that a part of the surface energy correction actually is a correction for remanent excess pore-water pressures.

Some of the problems discussed in the foregoing paragraphs may be said to be of academic rather than practical interest, because the full surface energy corrections for shear tests on clays usually are relatively small at the moment of failure, although they may attain appreciable magnitudes before failure.

The writer has not yet reached definite conclusions concerning the proper accounting for the surface energy corresponding to volume changes during shear tests on clays. Further investigations are needed concerning the magnitude of remanent pore-water pressures in the zone of failure during slow drained tests, and also for clarification of the mechanics and intrinsic forces involved in the consolidation and swelling of clays, caused directly or indirectly by shear stresses and strains.

Components of the Shear Strength

As indicated in the foregoing discussion and as shown in Fig. 6, the measured shear strength cannot be expressed as a unique function of the effective normal stress and the preconsolidation pressure. The writer's investigations show that the shear strength of remolded, saturated, and normally consolidated clays can be expressed either as a function of the effective normal stress, σ_f' , or as a function of the void ratio at failure, e_f . Furthermore, the shear strength of these clays in states of both normal consolidation

and simple and cyclic overconsolidation can be expressed by a combination of the two functions. It has been suggested that the components of the shear strength corresponding to the two functions be called the "stress component" and the "void ratio component," and these terms are quite logical and attractive. Nevertheless, the writer prefers in this paper to retain the established terms "effective friction component" and "effective cohesion component" in connection with extensions of the Coulomb condition of failure, and the two components are designated by τ_ϕ and c_e . The terms "stress component" and "void ratio component" may be preferable when the failure conditions are expressed in terms of the principal stresses or functions thereof. As discussed in the foregoing subsection, consideration of a "dilatation component" or "surface energy correction" has been suggested in recent papers. This component is designated by τ_d , and the measured shear strength, τ_f , may then be expressed by

$$\tau_f = \tau_\phi + c_e + \tau_d \tag{13-A}$$

Most cohesive soils possess an apparent structural viscosity and their deformations are of a visco-elastic character. The corresponding strength component may be called the "viscous component," but factors other than viscosity seem to be involved, and the more inclusive term "rheological component" and the notation c_v are proposed. It will be assumed that c_v forms a part of the effective cohesion component, c_e , because the effective friction component, τ_ϕ , of a remolded clay does not seem to be affected by the increased rate of deformation after failure, provided the soil structure is not changed; see Section 8. However, this assumption is in need of further experimental corroboration. The value of c_v converges on zero with increasing time or decreasing rate of deformation, whereas c_e at the same time approaches an ultimate value, c_u , which may be called the "ultimate cohesion component." By definition the following relation exists at any given test duration or rate of deformation

$$c_e = c_u + c_v \tag{13-B}$$

Eq. 13-A may then be written in the following rearranged form

$$\tau_f - \tau_d = \tau_\phi + c_e = \tau_\phi + c_u + c_e \tag{14}$$

where the terms on the left side represent the external energy and those on the right side, the internal energy.

For the purpose of definition and experimental determination of the individual components, the basic assumption is made that the cohesion and rheological components are constant when (1) the void ratio or water content of saturated clays is constant, (2) the rate of deformation or test duration is constant, and (3) there is no significant difference in the geometric structure of the clays during a given test series; see Section 1. The probability that the latter condition is fulfilled is enhanced by the observation that changes in geometric structure during strength tests tend to eliminate initial differences and to produce similar structures at the moment of failure; SEED and CHAN (1959). A shear strength diagram for constant void ratio is shown in Fig. 9 and illustrates simplified relations and definitions of the abovementioned strength components. As suggested by TERZAGHI (1938), this diagram is obtained by selecting points with the same void ratio or water content at failure on the normal consolidation, rebound, and reloading branches of a diagram similar to that shown in Fig. 6. Further details of the method are shown in Fig. 23.

The surface energy component, τ_d , is defined by Eqs. 11 and 12 and is introduced here in order to demonstrate its influence on the other components, but reference is made to the comments thereon in the foregoing section. The dilatation component τ_d is subtracted from the shear strengths, τ_f , obtained in the tests and represented by the shear strength line AB in Fig. 9. The corrected shear strength line, A'B', corresponds to the left side of Eq. 14. The general effect of consideration of the surface energy component is a slight increase of the inclination of the shear strength line and a small decrease of its zero intercept. According to experiments by GIBSON (1953), introduction of the full surface energy correction causes an increase of 1 to 2 degrees of the inclination of the shear strength, but the influence may be greater in case of undisturbed clays. The value of τ_d decreases with increasing test duration and is probably zero for tests of very long duration. It is also zero for all undrained or constant volume tests.

The effective friction component, τ_ϕ , is a function of the effective stress, σ_f' , and is defined by the shear strength line obtained when σ_f' is varied while the cohesion and rheological components remain constant and corrections are made for variations of the surface energy component, if any. The shear strength line thus obtained is usually straight, and its angle of

inclination, ϕ'_e , may be called the effective angle of internal friction. The friction component is then expressed by

$$\tau_\phi = \sigma'_f \tan \phi'_e = (\sigma_f - u) \tan \phi'_e \qquad (15)$$

where the pore-water pressure, u , appears as a part of the friction component and may be a time-dependent variable. The angle ϕ'_e depends on the composition of the clay and possibly also on the orientation of the flaky mineral particles or zones of stratification with respect to the surface of failure, but ϕ'_e has been found to be practically independent of the void ratio. Until further test data become available, it is assumed that ϕ'_e is independent of time or the rate of deformation.

The effective cohesion component, c_e , represents the strength caused by the intrinsic forces, and for the purpose of this paper it is defined as the zero intercept of the above-mentioned corrected shear strength line, A'B'. The effective cohesion can be expressed as a function of the void ratio, or the water content in case of fully saturated clays; but the coefficients in this function vary with the soil constituents, including the ions in the pore water or adsorbed on the surface of the clay particles, with the geometric structure of the clay, and possibly also with temperature, although the latter may primarily influence the rheological component. It should be noted that the void ratio and geometric structure referred to above are those existing at the moment of failure.

The ultimate cohesion component, c_u , is the intransient part of the effective cohesion component or the value which c_e approaches with increasing time or decreasing rate of deformation. Methods for estimating c_u are discussed in Section 7. This component can be expressed as a function of the void ratio, and the coefficients in this function vary with the constituents and geometric structure of the clay.

The rheological component, c_v , is the transient part of the effective cohesion component or the intercept OB' in Fig. 9, and it decreases to zero with increasing time. Methods for estimating c_v are discussed in Section 7. It is assumed that the rheological component is a function of the void ratio and the test duration or the rate of deformation, and the coefficients in this function vary with the constituents and geometric structure of the clay and with temperature. It should be noted that the decrease in strength of an undrained test specimen in part is caused by an increase in pore-water pressure

with time; BJERRUM, SIMONS, and TORBLAA (1958). Such an increase in pore-water pressure causes a decrease of the effective friction component, Eq. 15, and the rheological component is in this case defined as the difference between the total decrease in strength and the decrease of the friction component.

This concept or definition of the rheological component is admittedly a practical expedient until results of further investigations of the rheological properties of clays become available. Modifications of the definitions of several components will be required in case the results of future research should show that the transient part of the shear strength affects not only the intercept but also the inclination of the shear strength line in Fig. 9.

The residual shear strength. Most clays decrease in strength after failure. The ultimate value of the strength, called the residual shear strength and designated by τ_r in Fig. 4, depends not only on the type, structure, and state of consolidation of the clay but also on the type of loading, rate of deformation, drainage conditions, and the elapsed time after failure; see Section 8. The residual strength of some clays is attained only after very large deformations, and its determination then requires use of special equipment, such as the torsion shear apparatus.

In the case of remolded clays, a decrease in shear strength after failure is primarily caused by a transient increase in pore-water pressure and a thixotropic loss in strength, which is regained in time upon cessation of the deformations. However, a permanent decrease or increase in strength may be caused by an increase or decrease in void ratio after failure. Most undisturbed clays are subject to both a transient and a permanent decrease in strength after failure. The permanent part of this decrease in strength is primarily caused by alteration of the soil structure.

Significance of the components. The components of the shear strength defined in the foregoing paragraphs are primarily parts of mathematical expressions of the results of shear tests, and they may be called physical or phenomenological components which have not yet been definitely identified with specific intrinsic forces. Some investigators have suggested that there is no essential difference between the physicochemical forces which cause "friction" and "cohesion" in remolded clays, in which case the cohesion may be viewed as the result of secondary and residual changes in the spacing and arrangement of the clay particles. It is possible that the result of further research into the physicochemical and rheological properties of clays may suggest

modifications of the definitions and/or introduction of other components. Consequently, the writer prefers to retain the terms "effective angle of internal friction" and "effective cohesion" rather than using the terms "true angle of internal friction" and "true cohesion." Further delimitations of the definitions are discussed in the sections dealing with the individual strength components.

3. TESTING PROCEDURES AND SOURCES OF ERROR

Properties and Preparation of Soils Tested

The writer's research dealt primarily with the shear strength of a calcareous, silty clay from the vicinity of Vienna, Austria. A minor series of tests was also performed on a fat, Tertiary clay from Rögle Klint on the shore of Little Belt, Denmark. The average index properties and coefficients of these clays are summarized in Table 1. Both clays are subject to appreciable thixotropic changes in strength, which were investigated by means of the liquid limit device and the Swedish cone test at water contents slightly greater and less than the liquid limit.

Table 1

Index Properties of Soils Used in Tests

Property	Vienna Clay	Little Belt Clay
Specific gravity of soil particles, g/cm^3	2.76	2.77
Liquid limit	47	127
Plastic limit	22	36
Plasticity index	25	91
Clay size fraction (<0.002 mm), %	23	77
B = compression index for $(e, \ln \sigma')$ plot	8.4	2.6
e_o = void ratio for $\sigma' = 1.0\ kg/cm^2$	0.84	2.07
c_v = coefficient of consolidation, cm^2/sec	145×10^{-6}	94×10^{-7}

Complete consolidation and permeability tests were performed with consolidometers, using a molding water content slightly greater than the liquid limit. The virgin branches of the pressure-void ratio diagrams were also determined by means of the shear boxes, duplicating the conditions under which the specimens for shear tests are consolidated. That is, molding water contents slightly lower than the liquid limit were used; test specimens were consolidated under various normal pressures, and the water contents in the center of the test specimens were determined upon completion of the consolidation. The results obtained for Vienna clay are shown in Fig. 10. The compression index B refers to the void ratio versus natural logarithm of pressure; see Fig. 18. Similar results were obtained for Little Belt clay, and it is to be

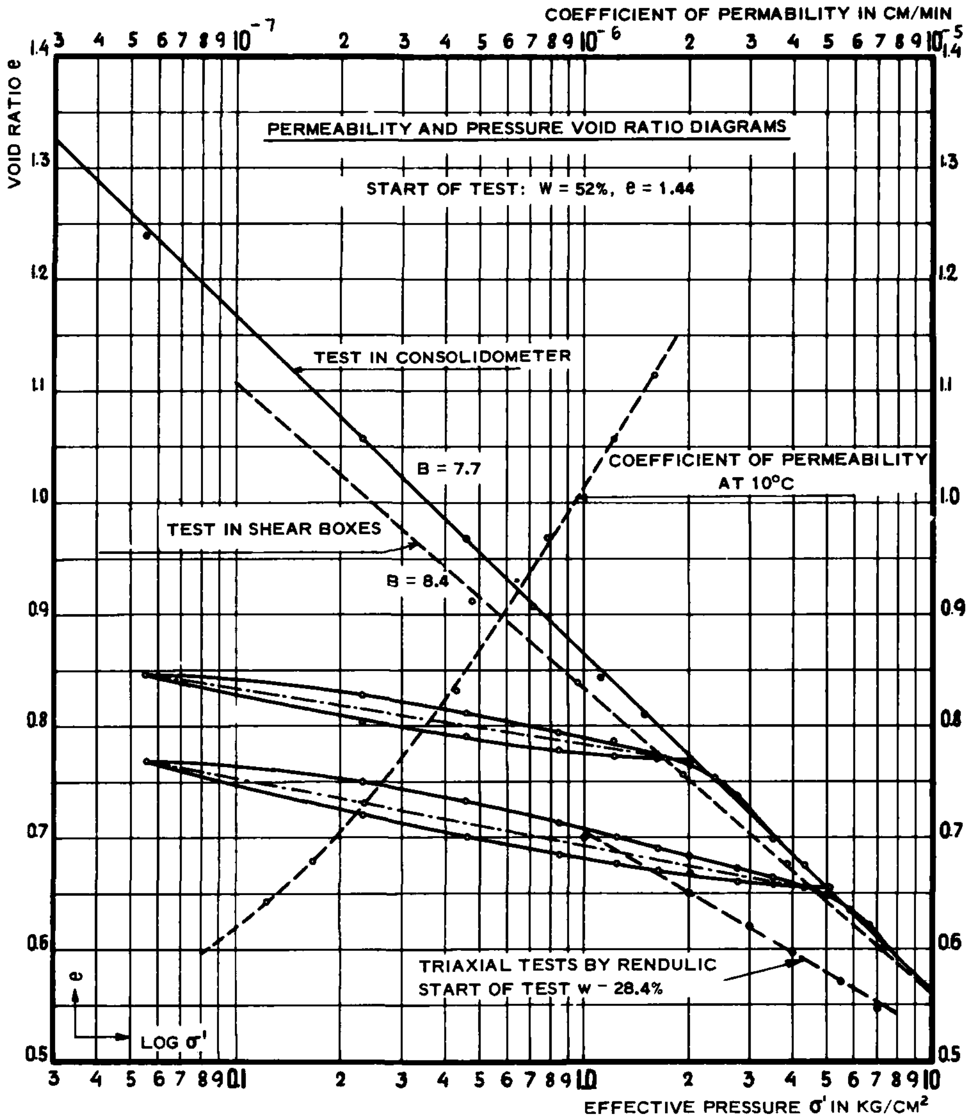


FIG. 10. CONFINED CONSOLIDATION AND PERMEABILITY TESTS ON VIENNA CLAY NO. V

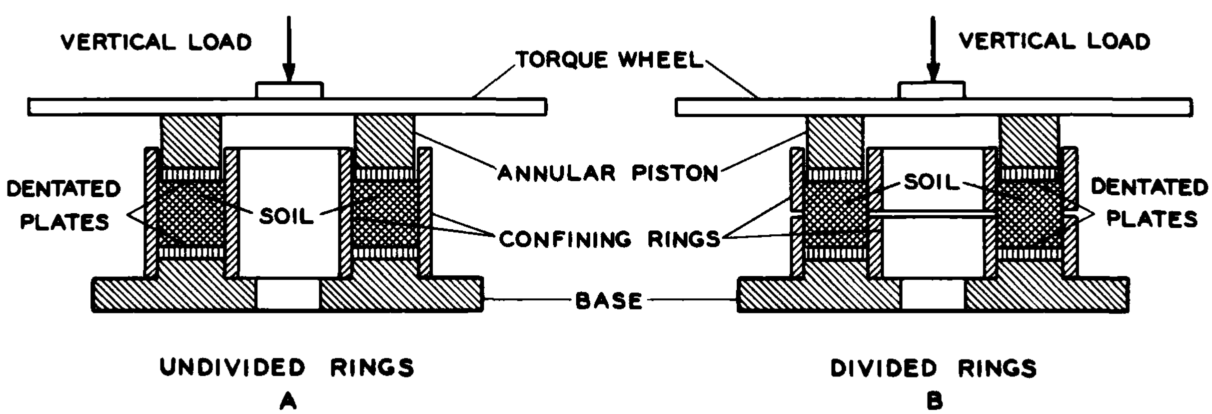


FIG. 11 - TORSION RING SHEAR APPARATUS

noted that the virgin branches of the $e - \log \sigma'$ diagrams are straight in all cases, and that the diagrams obtained by tests in the shear boxes lie below those obtained by means of the consolidometers because the molding water content was greater in the latter case.

The soils were thoroughly remolded at a water content slightly lower than the liquid limit, and each batch of remolded soil was stored for three to four weeks before being used in order to obtain a more uniform distribution of water content. Portions of a batch removed for preparation of test specimens were again remolded to remove the thixotropic regain of strength during storage. Nine batches of Vienna clay were prepared during the research, and there were slight differences in shear strength characteristics of the individual batches; the minimum value of $\tan \phi'_s$ was 0.485 and the maximum value 0.514, which must be taken into consideration when comparing results of tests on material from different batches. The available amount of Little Belt clay was small, and batch III consisted of clay which was remolded again after being used for the primary series of tests and then utilized for supplementary tests.

Shear Testing Equipment

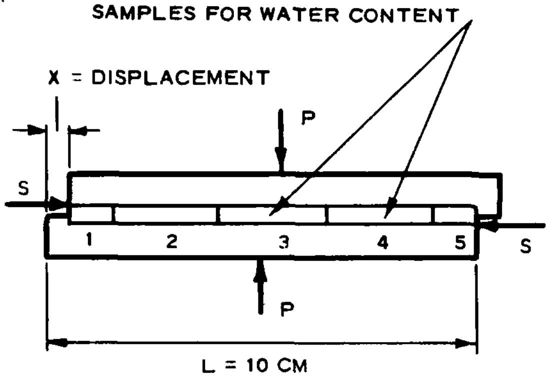
The principal strength tests were performed by means of the Krey direct shear apparatus and the Terzaghi shear boxes accommodating a 10-cm-square test specimen. Investigations of the plastic deformations before failure and the changes in shear strength after failure were made with a torsion shear apparatus and ring-shaped test specimens having an outside diameter of 11.95 cm and an inside diameter of 5.95 cm, Fig. 11. This equipment is described in detail in earlier papers; HVORSLEV (1937 to 1939). Both types of equipment were designed for use of incremental stress loading. A torsion shear apparatus with both controlled stress and controlled strain types of loading was later built by the USAE Waterways Experiment Station; HVORSLEV and KAUFMAN (1952). The torsion shear apparatus, Fig. 11, can be used either with divided confining rings or with solid confining rings. In the first case, failure occurs at midheight of the test specimen, as in the usual box shear test. With solid rings, failure occurs a short distance below the upper loading plate, and this arrangement was used for most tests of long duration since it eliminates the stress concentration at and possibility of leakage through the joint between the upper and lower rings in the arrangement with divided confining rings.

Test Specimens and Failure Deformations

Investigation of the distribution of water contents in normally consolidated test specimens showed that the water content is lower in the center than near the top and bottom surfaces. This distribution may be caused either by absorption of water from the porous stones during dismantling of the equipment at completion of a test and/or by a concentration of stresses and strains in the center of the test specimen. Special consolidation tests in the shear boxes indicated that the water content near the top and bottom surfaces of a normally consolidated test specimen may be increased by an amount corresponding to 0.5 to 1.5 per cent of the dry weight of the soil during the dismantling of the apparatus. The thickness required to eliminate post-test changes in water content at the center of the test specimen depends on the consolidation characteristics of the soil. Specimen thicknesses chosen on the basis of these results were 2.2 to 2.4 cm for Vienna clay and 1.8 to 2.0 cm for Little Belt clay.

Data obtained in tests with specimens of various thicknesses show that the horizontal and vertical deformations during a shear test do not increase linearly with the thickness of the test specimen. It is estimated that in box shear tests the effective thickness of a 2-cm-thick test specimen is only about 1.5 cm. The shear deformations of a thin test specimen are more uniformly distributed than those of a thick specimen, but the water content in the failure zone of a thin test specimen cannot always be determined with adequate accuracy because of absorption of water during dismantling of the apparatus.

The shearing forces were transmitted to the test specimen through dentated porous stones, with teeth 2.6 mm high and spaced 4.6 mm apart. The internal deformations in a test specimen were delineated by coloring thin vertical zones during consolidation and slicing and drying the test specimen after completion of the shear test. Fig. 13-A shows the pattern of internal deformations for a shear displacement of 4 mm. The displacement at failure was 3.5 mm, and it was estimated that the zone of nonuniform deformations or progressive failure then extended over a 1.5-cm-wide zone at the ends of the test specimen. Progressive failure also occurs in a zone along the sides of the test specimen, and colored sections near the sides indicated that this zone was about 0.75 cm wide. Fig. 13-B shows the deformations of a similar test specimen after a shear displacement of 8.5 mm, and it may be noted that inclined planes of failure now have been formed, probably because the lateral



EFFECTIVE AREA
$A_e = L\,(L - X)$

$$\sigma'_f = \frac{P + \Sigma P}{A_e} \qquad \tau_f = \frac{S + \Sigma S}{A_e}$$

FIG. 12. STRESSES AND WATER CONTENTS - DIRECT SHEAR TESTS

A - DISPLACEMENT X = 4.0 MM

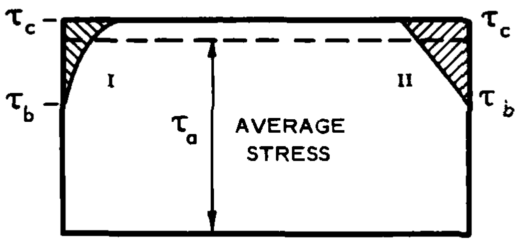
B - DISPLACEMENT X = 8.5 MM

NORMAL STRESS σ'_f = 2.0 KG/CM^2

DURATION T_s = 88 MIN

FAILURE DISPLACEMENT 3.5 MM

FIG. 13. INTERNAL DEFORMATIONS IN DIRECT SHEAR TESTS



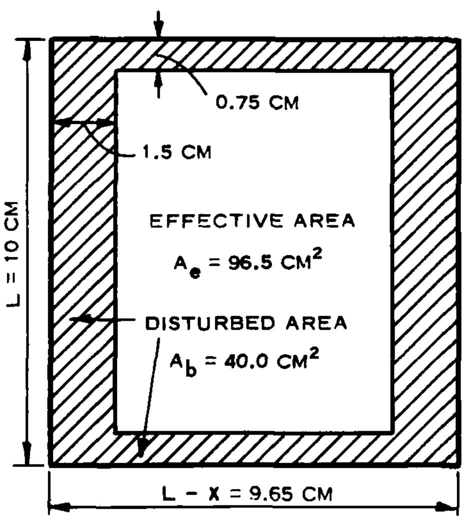
A

CASE I - PARABOLIC VARIATION

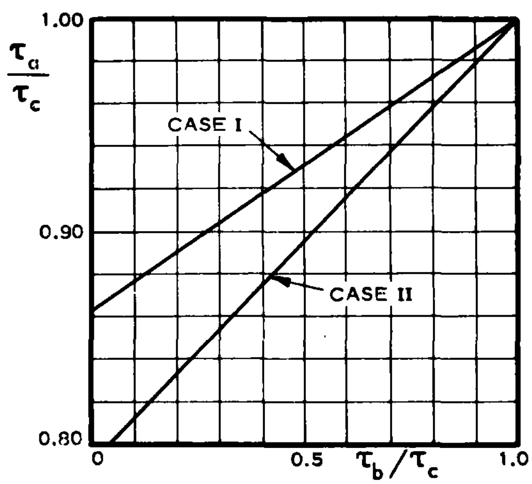
$$\frac{\tau_a}{\tau_c} = 1 - \frac{1}{3}\left(1 - \frac{\tau_b}{\tau_c}\right)\frac{A_b}{A_e}$$

CASE II - LINEAR VARIATION

$$\frac{\tau_a}{\tau_c} = 1 - \frac{1}{2}\left(1 - \frac{\tau_b}{\tau_c}\right)\frac{A_b}{A_e}$$



B



C

FIG. 14. INFLUENCE OF PROGRESSIVE FAILURE IN DIRECT SHEAR TESTS

confining stress at the upper and lower rear ends of the test specimen had been decreased or eliminated during the large deformations after failure. Such inclined zones of failure were much less pronounced or absent for strongly overconsolidated test specimens.

Variations in water contents of the soil near the teeth were investigated in special tests with porous stones having very large teeth. Corrections were made for water absorbed during dismantling, and it was found that the soil around the points of the teeth was consolidated to considerably lower water contents than the average for the test specimen. This zone of excess consolidation extended to a height approximately equal to half the distance between the teeth.

Tests were also made with the induced shear zone slightly above the teeth of the lower porous stone. The shear strengths obtained were nearly identical with those for the normal test arrangement when the test specimens were normally consolidated, but they were 8 to 10 per cent lower than those obtained with the standard arrangement when the specimens were strongly overconsolidated. It was also found that failure occurs outside the abovementioned zone of excess consolidation near the teeth. These results were confirmed by tests with the torsion shear apparatus. This difference in strengths obtained with the induced failure zone in the center of the test specimen and near the teeth of the dentated stones must be taken into consideration when comparing test data obtained in box shear tests with those obtained in torsion shear tests with solid confining rings.

Finally, tests were made with nondentated porous stones having either a medium coarse- or a medium fine-grained structure. The results obtained with coarse-grained stones were nearly identical with those obtained with dentated stones, but the shear strengths obtained with fine-grained stones were slightly lower, and there were indications of slippage at the surface of the fine-grained stones. These tests were made with normally consolidated test specimens, and a greater amount of slippage can be expected in tests with strongly overconsolidated test specimens, for which the normal stress is small compared to the shear strength. Finely dentated stones will prevent such slippage and are preferable to stones with relatively large teeth.

Calibration and Sources of Error

The calibration of the box and torsion shear test equipment covered not only lever ratios and mechanical friction in the equipment and dials,

sidewall friction for various stress conditions, but also the normal stresses and friction acting on the displaced ends of the box shear test specimen. The resulting corrected normal force and shear force were applied to the effective cross-sectional area at failure, A_e in Fig. 12, and the errors in the average values of σ'_f and τ_f thereby obtained are believed to be smaller than 0.01 kg/cm^2 or 1.0 per cent of the stress, whichever is larger.

The influence of progressive failure on the strength obtained by box shear tests may be estimated as shown in Fig. 14. As mentioned above, the disturbed zone at the ends of the test specimen has a width of approximately 1.5 cm, and that along the sides a width of about 0.75 cm. The area of the disturbed zone is then 40 cm^2, and the total effective area 96.5 cm^2 for a displacement of 3.5 mm at failure. The minimum value of the shear strength is designated by τ_b , the maximum value by τ_c , and the average value by τ_a . The variation of strength within the disturbed zone may follow a parabolic curve, Case I, or be linear, Case II. The relation between τ_a/τ_c and τ_b/τ_c may then be determined as shown in Fig. 14-C. For Vienna clay, τ_b is approximately equal to the residual strength or $\tau_b/\tau_c = 0.80$ (see Fig. 35-A) for which τ_a/τ_c is 0.96 to 0.97. The residual strength of Little Belt clay is not reached before very large displacements have taken place (see Fig. 35-B) and τ_b/τ_c is estimated to be about 0.60 for which τ_a/τ_c is 0.92 to 0.94. Corrections were not made for the influence of progressive failure in the box shear tests, and the values of τ_f , determined as shown in Fig. 12, represent the average values, τ_a , in Fig. 14.

A theory for determination of the stress distribution in the ring-shaped test specimen of a torsion shear test is presented in previous publications, HVORSLEV (1937, 1939). It was found that the maximum shear strength at failure is 0 to 3 per cent higher than the average maximum shear strength, depending upon the shape of the stress-strain curve. This theory does not consider the influence of stress concentration at the joint between the upper and lower rings or at the teeth of the porous stones. As mentioned earlier, values of the shear strength obtained by torsion shear tests agree well with those obtained by box shear tests in case of normal consolidation but are 8 to 10 per cent lower for strongly overconsolidated test specimens.

The above-mentioned influence on nonuniform stress distribution and progressive failure is to a large extent eliminated in a shear box developed by ROSCOE (1953). Test specimens in most direct box and torsion shear equipment so far developed are confined laterally and are subject to the influence of

sidewall friction. Corrections can be made for the influence of the sidewall friction, but the magnitude of the lateral stresses cannot be controlled. It was found in box shear tests with normally consolidated Vienna clay that the lateral pressure corresponds to a coefficient of earth pressure at rest, $K_o = 0.67$. However, the value of K_o increases with the degree of overconsolidation, because of residual lateral pressures, and the total lateral pressure may be greater than the vertical normal pressure for strongly overconsolidated test specimens. These large lateral pressures have some influence on the void ratios, the failure conditions, and the width of the shear strength hysteresis loop, Fig. 6.

Several investigators have proposed that the confining rings in torsion shear test equipment be eliminated and that the test specimen be covered with a thin rubber or plastic membrane or with grease and subjected to known lateral pressures as in a triaxial compression test. It may be possible to find membranes or coatings which are satisfactory for this purpose, but the test specimen will still be subjected to lateral restraint by the end plates. On the other hand, at short distances from the end plates, the test specimen may undergo changes in outside and inside diameters during a test. These changes are difficult to measure and have a significant influence on the torsional shear resistance of the specimen.

Testing Procedures

Material from the main supply or batch of remolded clay was again remolded and placed in the shear boxes to a thickness which would produce the desired thickness of the test specimen after consolidation. The normal loads were increased in increments similar to those used in standard consolidation tests. About one week was allowed for normal consolidation, two weeks for simple overconsolidation, and three weeks for cyclic overconsolidation. The degree of consolidation thereby attained was well beyond that corresponding to the end of the primary consolidation. The normal loads were removed for a few minutes while transferring the shear boxes from the consolidation loading bench to the shear testing apparatus. This transfer was made six to twelve hours before starting the actual shear test.

The shear loads were increased in increments which varied from 5 per cent of the estimated failure load at the start of a test to 1 or 2 per cent of the estimated failure load near the end of the test. The time interval between load increases for slow tests was varied from about 10 minutes at the

start to 30 minutes or several hours near the end of a test. An example of a time-loading curve is shown in Fig. 15, where T_f is the actual duration of the test. The equivalent duration, T_s, for a constant rate of load increase was computed by the geometric condition that the area OBT_s is equal to the area OAT_f, but it is believed that the actual equivalent test duration, producing the same degree of consolidation, is greater than the values of T_s thus computed.

The equipment was dismantled as quickly as possible upon completion of a test. A strip of soil 2 cm wide and 3 mm thick was cut from the center of the test specimen. This strip was divided into five sections, and the water content was determined for each section, Fig. 12. The water content in the failure zone was computed as the average of the water contents of sections 2, 3, and 4, since the water content of sections 1 and 5 often was changed by absorption of water during dismantling of the equipment.

Influence of the Duration of Tests

The influence of the test duration on the shear strength of Vienna clay was investigated for various consolidation conditions and thicknesses of the test specimen. Examples of the results obtained are shown in Fig. 16. Shearing stresses cause development of positive pore-water pressures in a normally consolidated soil. These pressures and the void ratio decrease whereas the friction and cohesion components increase, and the rheological component decreases with increasing duration of the test. It may be noted that the shear strength decreases slightly when T_s is increased from about 1000 minutes to 2320 minutes. Another series of tests, performed with the torsion shear apparatus, shows a decrease in strength of only 1 per cent when T_s is increased from 1300 minutes to 36 days. The results of tests on strongly overconsolidated Vienna clay are shown in the upper part of Fig. 16. In this case the shearing stresses caused development of negative pore-water pressures, which were equalized with increasing duration of the tests. Consequently, the friction, cohesion, and rheological components all decreased with increasing duration of the test. The maximum value of T_s in this test series was only 515 minutes, and a further slight decrease in strength would undoubtedly have been found for larger values of T_s.

Based on the test results shown in Fig. 16, an equivalent test duration T_s = 600 to 800 minutes was used for slow, drained tests on Vienna clay. The actual degree of consolidation obtained cannot be determined directly from the

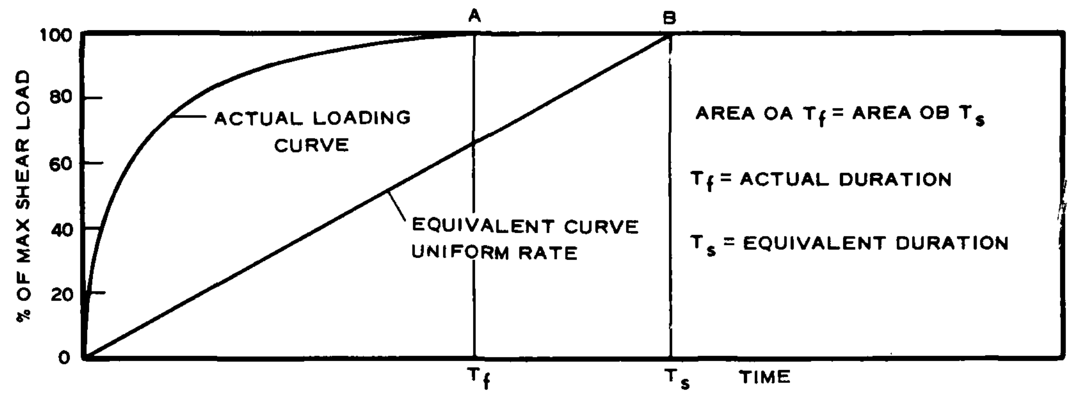


FIG. 15. LOADING PROCEDURE AND EQUIVALENT TEST DURATION

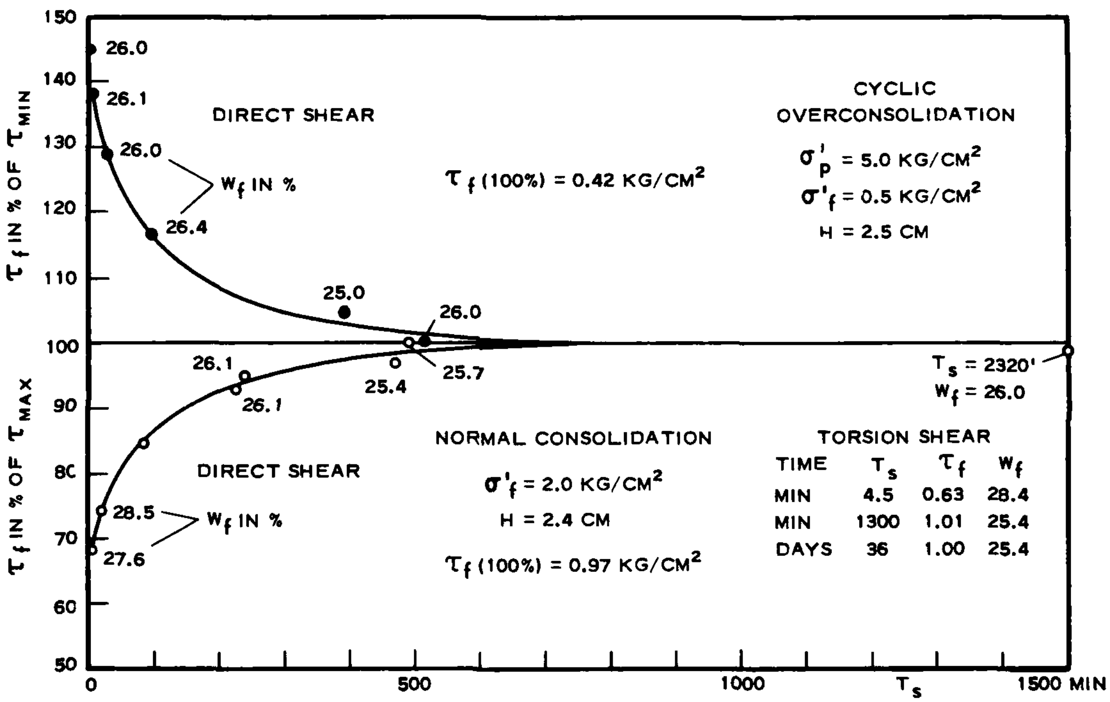


FIG. 16. INFLUENCE OF TEST DURATION FOR VIENNA CLAY

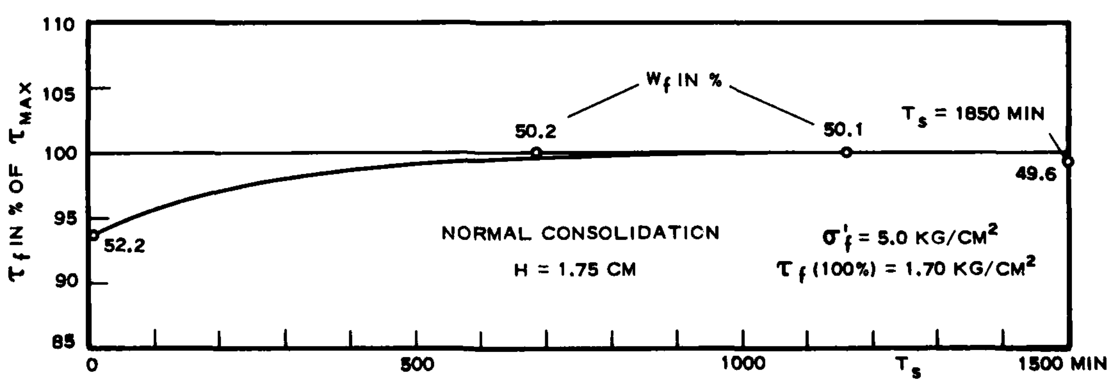


FIG. 17. INFLUENCE OF TEST DURATION FOR LITTLE BELT CLAY

test results because of the influence of the rheological component combined with slight variations or irregularities in water contents at failure. However, GIBSON and HENKEL (1954) have developed a very useful theory for determination of the required duration of drained tests or, conversely, the degree of consolidation obtained for a given test duration. For T_s = 700 minutes, H (thickness of the test specimen) = 2.2 cm, and c_v (the coefficient of consolidation) = 87×10^{-4} cm^2/min for normal consolidation, the theory furnishes the following values of the degree of consolidation:

Average for entire test specimen, $U_c = 1 - (H/2)^2/3c_vT_s = 0.944$

At center of test specimen, $U_c = 1 - (H/2)^2/2c_vT_s = 0.901$

As mentioned by the authors, the theory furnishes results which are on the safe side because it is based on the assumption that the shear stresses and deformations are uniformly distributed over the entire height of the test specimen. The writer found that the deformations and changes in water content are greatest in the center, and that the effective thickness of a fairly thick test specimen is considerably smaller than the actual thickness, and it is believed that the degree of consolidation attained in slow, drained tests on Vienna clay was at least 95 per cent in case of normal consolidation and greater for overconsolidation, because the coefficient of consolidation then is much greater than for normal consolidation.

The supply of Little Belt clay was very limited, and only a few tests for determination of the influence of the test duration were performed. The results of some of these tests are shown in Fig. 17. Other tests show that an increase of T_s from 970 minutes to 2600 minutes causes less than 1 per cent increase in shear strength. On the basis of these results it was decided to use T_s = 800 to 1600 minutes for slow, drained tests on Little Belt clay. However, considering the values of the consolidation coefficient, Table 1, and the thickness of the test specimen, H = 1.8 to 2.0 cm, the duration of slow tests on Little Belt clay should be about twelve times that used for Vienna clay in order to obtain the same degree of equalization of excess pore-water pressures at failure. This apparent contradiction between test results and theory was not discovered before all tests had been completed, and a definite explanation thereof requires additional tests. The influences of the rheological component and the remaining excess pore-water pressures at failure undoubtedly compensate each other to some extent, but it is not certain that the influence of the rheological component alone can explain the test results

shown in Fig. 17. Reference is made to similar results recently obtained by BJERRUM, SIMONS, and TORBLAA (1958).

Summary: Reliability of Test Results

It is believed that errors in measured shear and normal stresses, adjusted in conformity with the calibration of the equipment, is less than 0.01 kg/cm^2 or 1.0 per cent of the stress. However, corrections were not made for the influence of progressive failure; see Fig. 14.

The dilatation or surface energy component was not considered because the concept of this component had not been developed at the time of the tests. Sufficient data for computation of the component are not available, but the component determined by Eq. 11 would be very small because of the loading procedure and relatively high rates of shear deformation at the time of failure.

The shear strengths determined include the rheological component, which is relatively small in the case of slow, drained tests on Vienna clay. Estimation of this component is discussed in Section 7.

It is believed that the degree of consolidation attained in slow, drained tests on Vienna clay was at least 95 per cent. The influence of the remaining excess pore-water pressures is in part compensated by neglecting the influence of progressive failure in evaluation of the test results.

In spite of the test data shown in Fig. 17, it is probable that the duration of the slow, drained tests on Little Belt clay was too short and that appreciable excess pore-water pressures existed at the moment of failure and affected the computed values of the strength parameters. However, the errors may be relatively small because of compensating factors, and the values obtained are probably significant in a qualitative sense.

4. CONSOLIDATION CHARACTERISTICS AND SHEAR STRENGTH

The Equivalent Consolidation Pressure

A change in void ratio or water content causes a change in the shear strength of a clay, and the consolidation characteristics of a clay form a part of a complete expression of the shear strength. The virgin branch of a semilogarithmic plot of the consolidation diagram is usually straight (Fig. 18) and can be expressed by the equation

$$e = e_o - C_c \log\left(\frac{\sigma'}{\sigma'_o}\right) \tag{16}$$

where C_c is the compression index and σ'_o is the consolidation pressure corresponding to e_o. Expressed in terms of natural logarithms and using the compression index B, originally defined by Terzaghi, the equation is

$$e = e_o - \frac{1}{B} \ln\left(\frac{\sigma'}{\sigma'_o}\right) \tag{17}$$

Eq. 17 was used in former papers and will also be used in the following developments since it simplifies the mathematical expressions.

The equivalent consolidation pressure, σ'_e, corresponding to the void ratio e is defined as the pressure σ'_e for a point on the virgin branch of the consolidation diagram with the ordinate e, or by rearranging Eq. 17

$$\sigma'_e = \sigma'_o \exp[B(e_o - e)] \tag{18}$$

The graphical determination of σ'_e is shown in Fig. 18 for the points F and G on the rebound and reloading branches of the diagram, but the method and Eq. 18 apply to any value of e. The substitution $w = e/G_s$ may be used for fully saturated clays, where G_s is the specific gravity of the soil solids. Introduction of σ'_e means that e or w is represented by a quantity with the same dimensions as τ_f and σ'_f, which facilitates graphical or analytical determination of the failure conditions for clays. Representation of e by σ'_e is to be considered as a mathematical expedient and not necessarily dependent on the actual pressure-void ratio diagram; it will be shown later that any line parallel to the virgin branch of the consolidation diagram, Fig. 18, can be used for determination of σ'_e. The graphical determination of σ'_e shown in Fig. 18 was first used by TERZAGHI and JANICZEK, see TERZAGHI (1931-B),

in their investigation of the unconfined compressive strength of clays.

The clay is in a state of overconsolidation when $(e\ ,\ \sigma')$ lies below the virgin branch of the consolidation diagram or when σ' is smaller than σ'_e , and the degree of overconsolidation is defined by

$$n_c = \sigma'_e/\sigma' \tag{19}$$

The largest effective consolidation pressure to which a soil sample or test specimen has been subjected, σ'_p in Fig. 18, is called the preconsolidation pressure or, according to CASAGRANDE and WILSON (1953), the prestress and

$$n_p = \sigma'_p/\sigma' \tag{20}$$

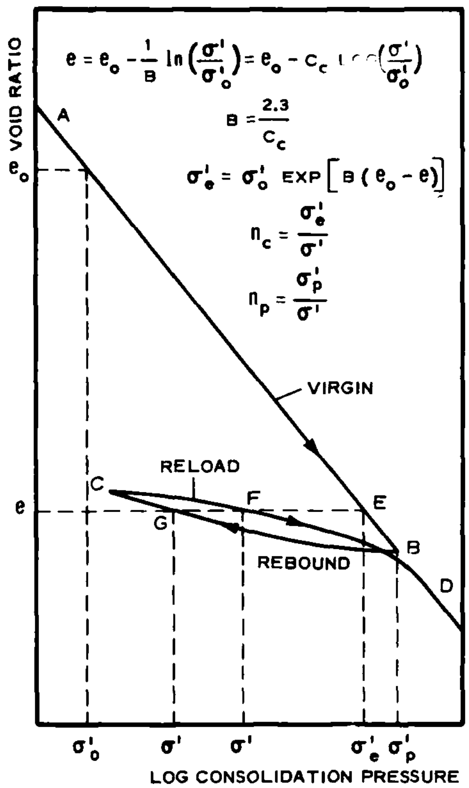
is the prestress ratio.

Graphical Presentation of Shear Test Results

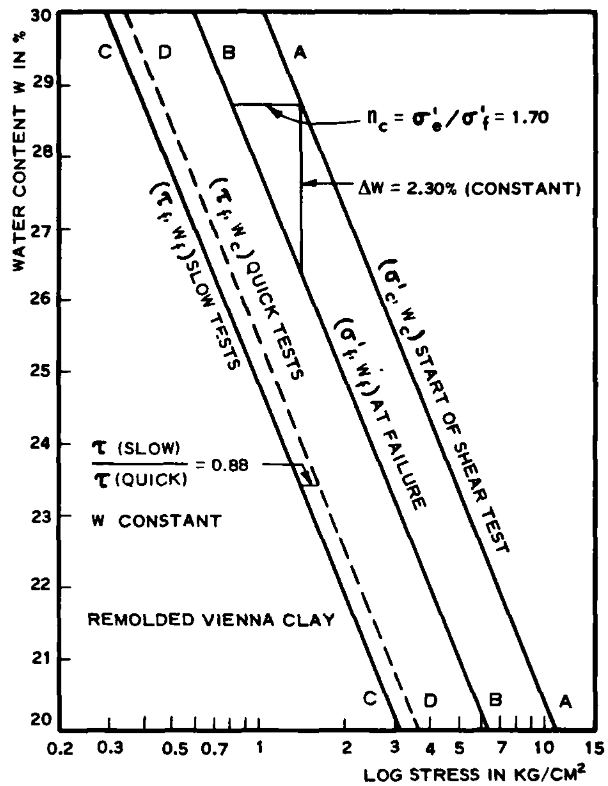
The principal results obtained in slow direct shear tests on Vienna clay and Little Belt clay are shown in Figs. 21 and 22. Some of the data for Vienna clay are also shown in Figs. 19 and 20 where water contents are plotted versus the logarithms of σ'_c , σ'_f , or τ_f . This form of presenting the results of shear tests was originated during the Cooperative Triaxial Shear Research Program of the Corps of Engineers, RUTLEDGE (1947), and it was later used by HENKEL (1958, 1959, 1960) in presenting results of strength tests on both normally consolidated and overconsolidated clays.

The writer used the equivalent consolidation pressure, σ'_e , to express the relation between void ratio and shear strength, and equivalent pressure curves for conditions at the start and at the end of the shear test are shown in Figs. 21 and 22. It should be noted that the equivalent pressure curves for void ratios at the end of the consolidation or start of the shear test proper are based on data obtained in standard consolidation tests, since consolidation tests in the shear boxes were confined to normal consolidation. As shown in Fig. 11, there is some difference in the position and slope of the virgin branches of the consolidation diagram obtained in consolidometers and shear boxes, primarily because of differences in the molding water content. These differences are to a large extent eliminated when the consolidation diagrams are transformed into equivalent pressure diagrams.

The shear strength line for normal consolidation is straight in case of Vienna clay and slightly curved for Little Belt clay. In discussions of the



EQUIVALENT CONSOLIDATION PRESSURE
FIG. 18



SHEAR STRENGTH AND WATER CONTENT –
NORMAL CONSOLIDATION
FIG. 19

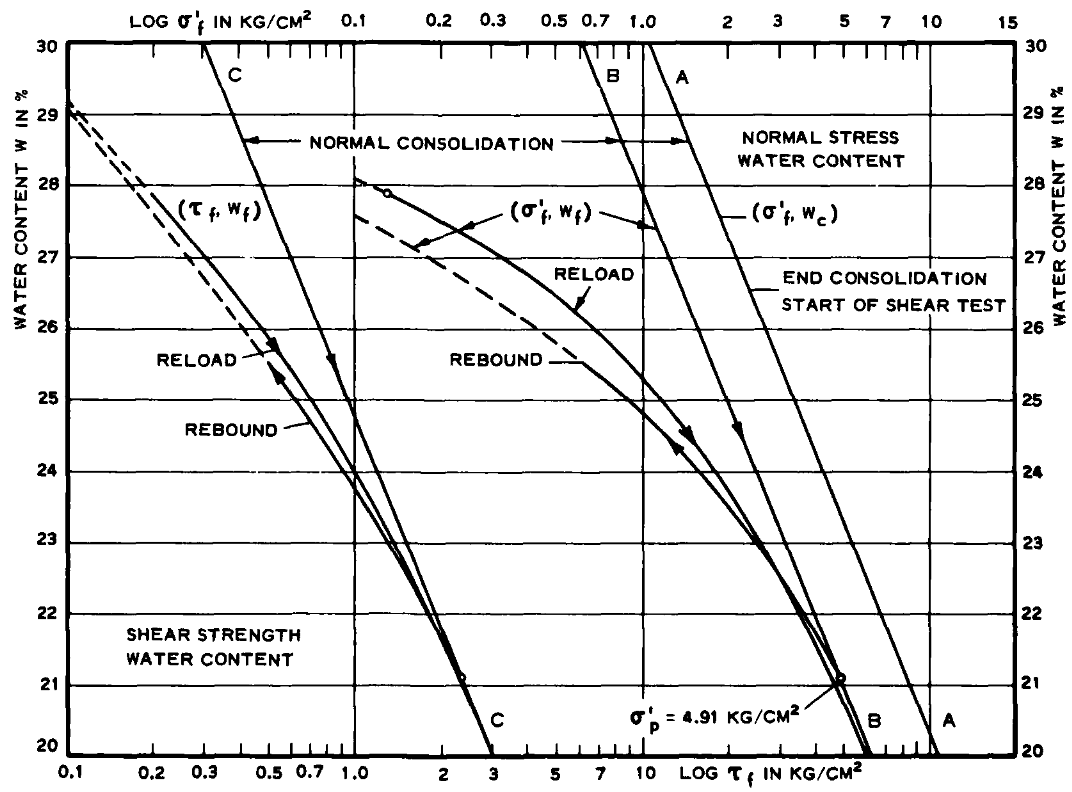


FIG. 20. SHEAR STRENGTHS AND WATER CONTENTS OF VIENNA CLAY

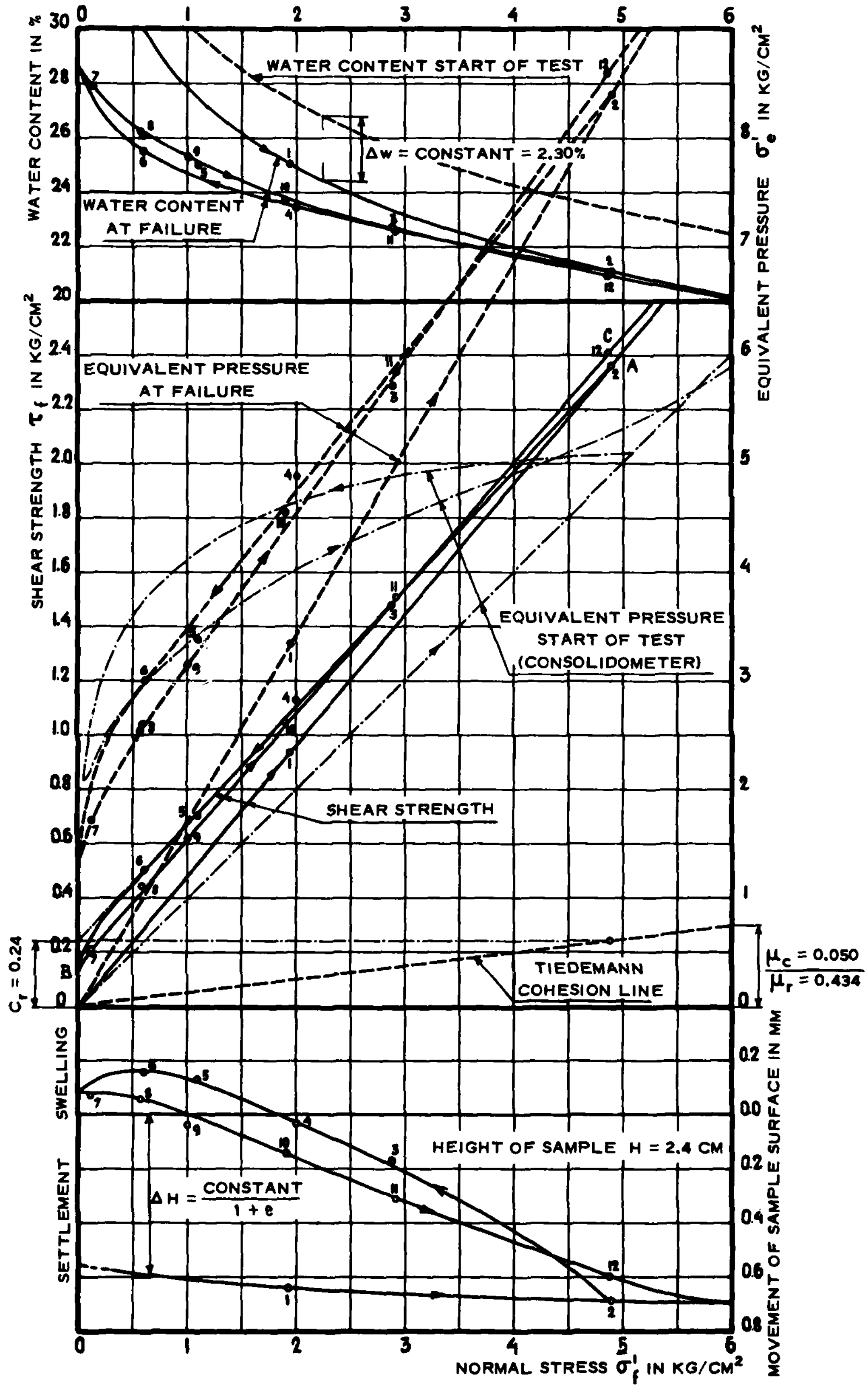


FIG. 21. RESULTS OF SLOW DIRECT SHEAR TESTS ON VIENNA CLAY V

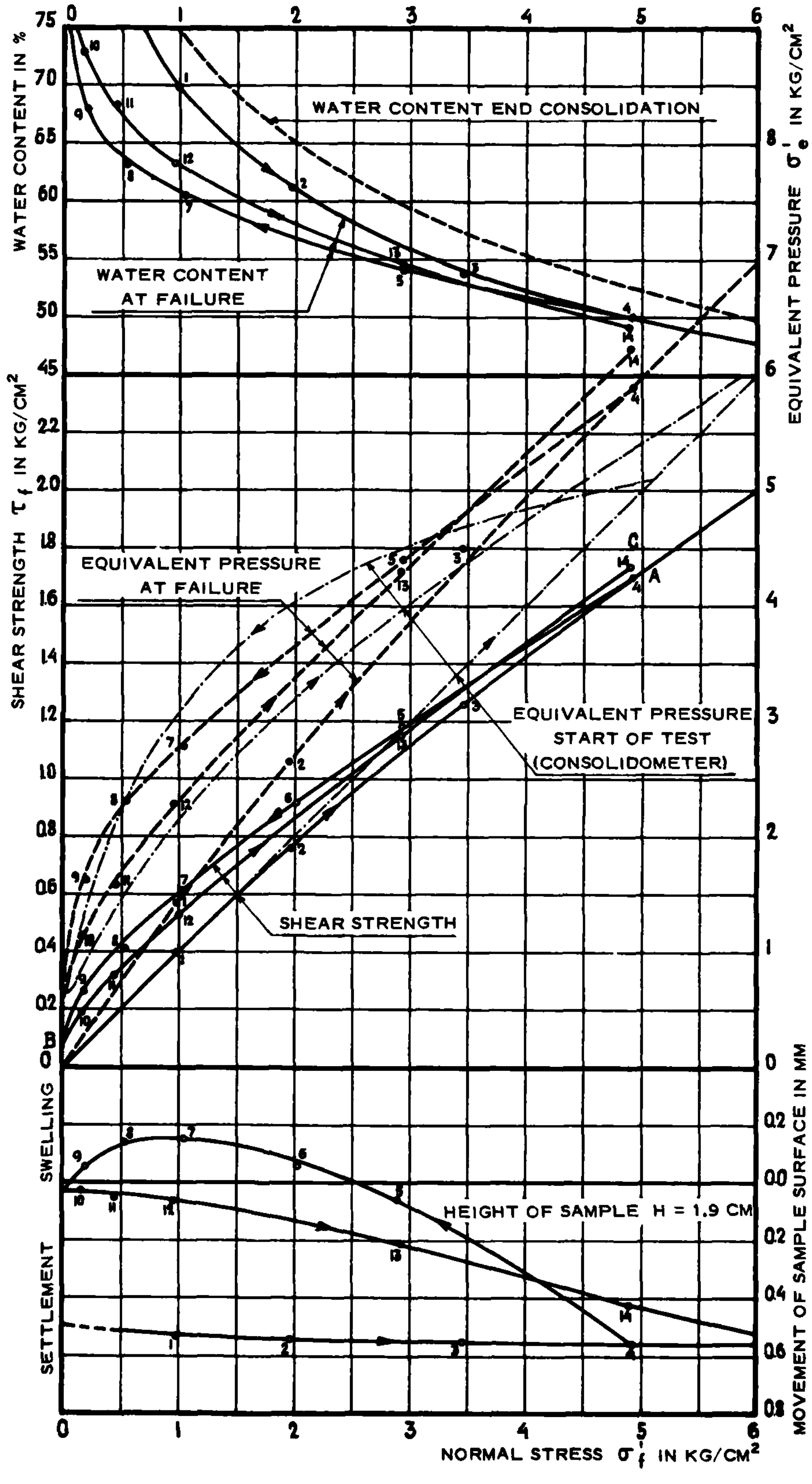


FIG. 22. RESULTS OF SLOW DIRECT SHEAR TESTS ON LITTLE BELT CLAY

writer's original papers it has been suggested that the curvature of the shear strength line for Little Belt clay is caused by insufficient duration of the tests. As indicated in Section 3, it is probable that the duration of tests on Little Belt clay was too short and that excess pore-water pressures existed at the moment of failure. However, the writer does not believe that this could cause the curvature of the shear strength line, since the coefficient of consolidation is practically constant for the entire range of pressures used in the tests. Furthermore, the shear strength lines for normal consolidation are straight for both slow and quick tests in case of Vienna clay but curved for Little Belt clay, Fig. 31, which indicates that the test duration does not influence the shape of these lines. In contrast thereto the results of consolidation tests on Little Belt clay are similar in form to those obtained for Vienna clay; that is, the virgin branches of the semilogarithmic consolidation diagrams are straight for both clays. The results of tests on Little Belt clay exhibit other anomalies which will be discussed later. It is difficult to remold a very fat clay completely, and the influence, if any, of inadequate remolding on the anomalies should be investigated.

Volume Changes During Shear Tests

As first shown by CASAGRANDE and ALBERT (1932), the void ratio or water content of normally consolidated clays decreases during a shear test. The movements of the surface of the test specimen and the relative position of equivalent pressure lines, shown in Figs. 21 and 22, indicate that both normally consolidated and slightly overconsolidated clays are subject to a decrease in volume during a shear test. Consequently, the shear stresses cause a temporary increase in pore-water pressures and the shear strength increases with increasing duration of the test when drainage is possible; see Figs. 16 and 17.

Other tests, HVORSLEV (1937, 1938), show that interruption of a very slow, drained shear test on normally consolidated Vienna clay and decrease of the shear stresses cause a slight additional decrease in void ratio. RENDULIC (1937) found that interruption of an undrained triaxial compression test on normally consolidated Vienna clay and decrease of the axial stress caused a slight increase instead of the expected decrease in pore-water pressure. That is, a decrease in shear stresses may cause a change in void ratio or pore-water pressure which is of the same sign but smaller magnitude than that caused by an increase in shear stresses. RENDULIC (1936, 1937) also observed

that the initial volume decrease of a normally consolidated clay during a triaxial compression test was reduced by a slight volume increase near failure. This reversal of the volume change near failure may have been caused by the use of a molding water content close to the plastic limit. The above-mentioned observations are discussed in greater detail in Section 9.

The test data in Figs. 21 and 22 show that the volume of strongly overconsolidated clays increases during a shear test. Consequently, the shear stress causes a temporary decrease in pore-water pressure and the shear strength of such clays decreases with increasing time, Fig. 16. BUISSON (1936) also found that the volume of certain dense clays increases during a shear test. It appears, however, that the sign of the volume change during shear depends on the stress history and prestress ratio rather than on the density or void ratio of the clay. At a certain prestress ratio, the shearing stress does not cause any change in volume or pore-water pressures, but this critical prestress ratio is quite different for simple overconsolidation and cyclic overconsolidation, and all cyclic overconsolidated test specimens of Little Belt clay were subject to a volume decrease during the shear tests.

The volume changes observed in direct shear tests are probably influenced to some extent by residual lateral stresses, but the general character of the results has been verified by other investigators and by triaxial tests on both remolded and undisturbed clays, and these results are important for estimating the short- and long-term shear strengths of clays.

The results of shear tests on normally consolidated test specimens of Vienna clay, Fig. 21, also show that the decrease in water content, Δw, or void ratio, Δe, during a shear test is constant and independent of the normal stress σ_f'. This means that the pressure-void ratio line for failure conditions is parallel to the virgin branch of the consolidation diagram, Fig. 19. These results were also obtained by RUTLEDGE (1947) and HAEFELI (1951) and have been verified by others. The degree of overconsolidation at failure, n_c, of originally normally consolidated clays can then be determined as follows. The void ratio at the end of the consolidation or start of the shear test is e_c, and that at failure is $e_f = e_c - \Delta e$. The normal stress remains constant during a drained shear test and can, according to Eq. 17, be expressed by

$$\sigma_f' = \sigma_c' = \sigma_o' \exp[B(e_o - e_c)] \tag{21}$$

The equivalent consolidation pressure at failure, as defined by Eq. 18, is

$$\sigma'_e = \sigma'_o \exp[B(e_o - e_f)] \tag{22}$$

and Eqs. 19, 22, and 23 then yield

$$n_c = \frac{\sigma'_e}{\sigma'_f} = \exp[B(e_c - e_f)] = \exp(B \cdot \Delta e) \tag{23}$$

The following values were obtained for Vienna clay, batch V, $B = 8.4$, $\Delta e = 0.0635$, and $n_c = 1.70$.

The numerical value of the change in water content or void ratio during shear tests on normally consolidated test specimens of Little Belt clay decreases slightly with increasing normal stress, and the above-mentioned relations for Vienna clay do not apply to Little Belt clay. HENKEL (1958, 1959) obtained similar results in drained triaxial compression tests on normally consolidated test specimens of Weald clay and London clay, which are comparable to Vienna clay and Little Belt clay. It was found that the change in void ratio, Δe , was a constant for Weald clay but decreased slightly with increasing consolidation pressure for London clay.

The Shear Strength of Normally Consolidated Clays

In the writer's original papers the shear strength of normally consolidated clays was treated as a special case of the equations for the shear strength of both normally consolidated and overconsolidated clays. However, the relation between shear strength and water content or void ratio for normally consolidated clays can be developed without special assumptions as long as the shear strength can be expressed by

$$\tau_f = \sigma'_f \tan \phi'_s = \sigma'_f \cdot \mu_s \tag{24}$$

the virgin branch of the consolidation diagram by Eq. 17, and the change in void ratio during the shear test, $\Delta e = e_c - e_f$, is independent of the normal effective stress, σ'_f . These conditions are not fulfilled by and the following derivations do not apply to Little Belt clay. According to Eq. 24

$$\frac{\tau_f}{\tau_o} = \frac{\sigma'_f}{\sigma'_o}$$

which inserted in Eq. 17 yields

$$\ln\left(\frac{\tau_f}{\tau_o}\right) = B(e_o - e_c) = B(e_o - e_f - \Delta e) \tag{25}$$

and

$$\tau_f = \tau_o \exp(Be_o) \exp(-Be_c) = C_1 \exp(-Be_c) \tag{26}$$

or

$$\tau_f = \tau_o \exp[B(e_o - \Delta e)] \exp(-Be_f) = C_2 \exp(-Be_f) \tag{27}$$

where C_1 and C_2 are constants for a given soil. These equations also apply to the results of slow undrained tests, for which $e_f = e_c$ and $\Delta e = 0$. The equations show that the shear strength of a normally consolidated clay can be expressed as a function of the water content at the start of the test or at failure, and that a plot of the logarithm of the strength versus water content forms a straight line, CC in Fig. 19, parallel to the virgin branch of the consolidation diagram. These results have also been obtained independently by RUTLEDGE (1947), HAEFELI (1951), and others. HENKEL (1958, 1959) performed slow triaxial tests on both drained and undrained test specimens of remolded and normally consolidated clays. It was found that the results of both drained and slow undrained tests can be presented by a single straight line corresponding to CC in Fig. 19. These results and other comparisons made by Henkel show that the prestress developed in undrained tests on remolded and normally consolidated clays has no effect on the failure conditions when the duration of the undrained tests is equal to that used in slow drained tests. However, prestressing may have considerable influence on the deformation characteristics of a clay.

The shear strength of a normally consolidated clay can also be expressed as a linear function of the equivalent consolidation pressure. Eqs. 19 and 24 yield

$$\sigma_f' = \sigma_e'/n_c \qquad \text{and} \qquad \tau_f = (\sigma_e' \tan \phi_s')/n_c \tag{28}$$

By inserting the expression for σ_e' in Eq. 18 and $\tau_o = \sigma_o' \tan \phi_s$, Eq. 26 is transformed into Eq. 27.

Quick direct shear tests on Vienna clay yielded a straight shear

strength line for normally consolidated test specimens, Fig. 31-A. Therefore, equations similar to those representing the results of slow tests can also be developed for the results of quick tests. That is, the shear strength obtained in quick tests can be expressed as a linear function of the total normal stress, σ_f , or the equivalent consolidation pressure, σ'_e , or as an exponential function of the water content. A plot of water content versus the logarithm of shear strength is a straight line, DD in Fig. 19, which lies slightly to the right of the line CC. That is, the quick tests yield shear strengths which are slightly greater than those obtained in slow tests for the same water content at failure. The difference represents the influence of the rheological component, and this difference can also be expressed as a function of the equivalent consolidation pressure or the water content or void ratio at failure. The influence of the rheological component is discussed in greater detail in Section 7.

5. THE EFFECTIVE FRICTION AND COHESION COMPONENTS

Theoretical Derivations

It has been shown above that the shear strength of a normally consolidated Vienna clay can be expressed as a linear function of the effective normal stress, σ'_f , or the equivalent consolidation pressure, σ'_e , or as an exponential function of the void ratio, e_f , or the water content, w_f , at failure. Consequently, the shear strength can also be expressed as a compound function of σ'_f and σ'_e or e_f . An inspection of Figs. 20 and 21 shows that the shear strength of overconsolidated test specimens of Vienna clay cannot be expressed as a unique function of either σ'_f , σ'_e , or e_f . However, the similarity of the shear strength and equivalent pressure diagrams in Figs. 21 and 22 suggests that the shear strength of both normally consolidated and overconsolidated clays may be expressed as a compound linear function of σ'_f and σ'_e . Hence it is postulated

$$\tau_f = \mu_e \sigma'_f + \kappa \sigma'_e \tag{29}$$

where $\mu_e = \tan \phi'_e$. The equation may also be written

$$\frac{\tau_f}{\sigma'_e} = \mu_e \frac{\sigma'_f}{\sigma'_e} + \kappa = \frac{\sigma'_f}{\sigma'_e} \tan \phi'_e + \kappa \tag{30}$$

This equation has the simple form of the Coulomb criterion, Eq. 3, and if the shear strength can be expressed mathematically by this equation, the points $(\sigma'_f/\sigma'_e\ ,\ \tau_f/\sigma'_e)$ should lie on a straight line with the angle of inclination ϕ'_e and an ordinate intercept κ . Fig. 24-A shows that points representing data obtained in tests on Vienna clay, batch V, form a straight line with very little scatter, even when certain tests have been duplicated because of minor irregularities in the first run of these tests. Similar results but slightly different values of ϕ'_e and κ were obtained in tests with Vienna clay, batch I.

The results obtained in tests with Little Belt clay are shown in Fig. 24-B, and the points also lie close to a straight line, but the scatter is greater than in the case of Vienna clay, and a line connecting the various points still forms a faint hysteresis loop and a separate line with a flatter slope for the points representing the state of normal consolidation. As mentioned before, it is probable that appreciable excess pore-water pressures

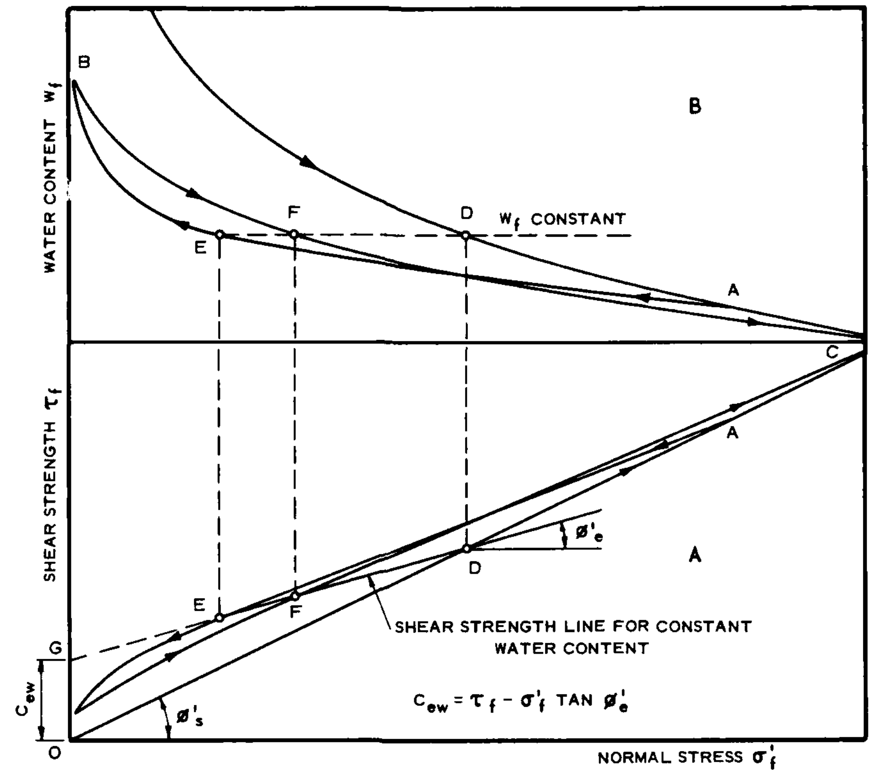


FIG. 23. SEPARATE DETERMINATION OF FRICTION AND COHESION COMPONENTS

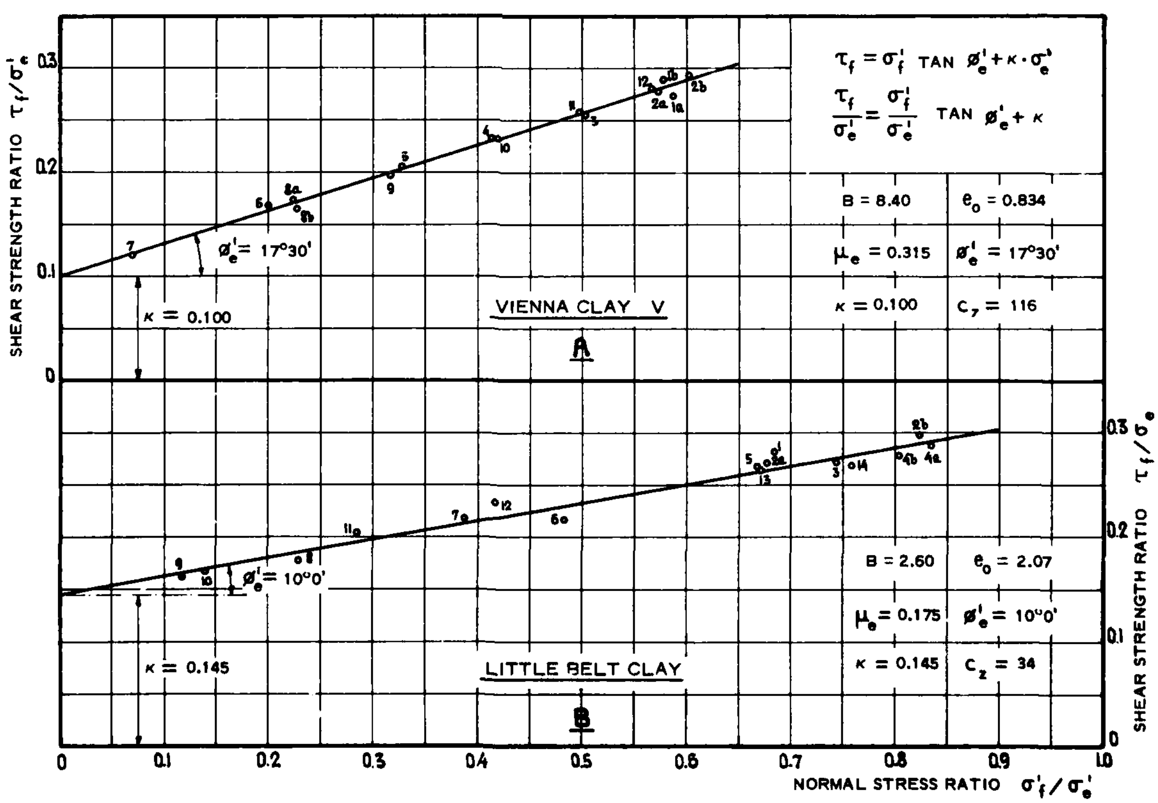


FIG. 24. COMBINED DETERMINATION OF FRICTION AND COHESION PARAMETERS

existed at the time of failure in drained tests on Little Belt clay, and the numerical results are influenced by partially compensating errors and cannot be considered fully reliable. However, the results indicate that the curvature of the shear strength line for normally consolidated Little Belt clay in part can be explained by the observation that the change in void ratio during the shear test decreases with increasing values of σ'_f .

By introducing the expression for σ'_e in Eq. 22 the last member of Eq. 29 can be written as follows,

$$\kappa\sigma'_e = \kappa\sigma'_o \exp[B(e_o - e_f)] = c_z \exp(-Be_f) \tag{31}$$

where

$$c_z = \kappa\sigma'_o \exp(Be_o) \tag{32}$$

is a coefficient which represents the value of the function or the effective cohesion component for zero void ratio, $e_f = 0$; see Fig. 26.* This is merely a mathematical interpretation of Eq. 32, and it is possible that the above-mentioned relations change when the void ratio approaches zero. Eq. 29 can then be written in the alternate form

$$\tau_f = \sigma'_f \tan \phi'_e + c_z \exp(-Be_f) \tag{33}$$

This equation shows that the shear strength can be expressed as a function of the effective normal stress on and the void ratio in the plane of failure at the moment of failure, and that this function is independent of the stress history of the clay.

It should be noted that Eq. 33 is a mathematical expression of the shear strength which is valid without special assumptions when the points (σ'_f/σ'_e , τ_f/σ'_e) lie on a straight line, as shown in Fig. 24. The first member on the right side of the equation is equivalent to the effective friction component, τ_ϕ , and the second member to the effective cohesion component, c_e , defined in Fig. 9. As previously mentioned, corrections were not made for the influence of the surface energy component, and it is assumed that the

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* The symbol ν in former papers is here changed to c_z because ν commonly is used as a symbol for the Poisson ratio.

rheological component in case of slow tests is negligible, or is included in the effective cohesion component. Special assumptions, discussed in connection with Figs. 9 and 23, are required when the component parameters are determined separately and also to explain the physical meaning of the components.

Introduction of the equivalent consolidation pressure, σ'_e , is a mathematical expedient, and any line parallel to the virgin branch of the consolidation may be used for this purpose. The change from one line to another merely causes the values of σ'_e to be multiplied by a constant; the slope of the lines in Fig. 24 will not be changed, but the value of the intercept, κ , will be divided by the same constant; hence the product $\kappa\sigma'_e$ remains constant. However, lines which are not parallel to the virgin branch of the consolidation diagram should not be used for determination of σ'_e , since this would cause the straight lines in Fig. 24 to become curved or change the apparent slope of the lines. In general, it may be most expedient to determine the values of σ'_e by means of the line which represents the water contents or void ratios at failure of normally consolidated test specimens, line BB in Fig. 19.

Eq. 29 may be written in two alternate forms when the clay is normally consolidated, in which case $\sigma'_e = n_c \sigma'_f$, according to Eq. 23, and

$$\tau_f = (\mu_e + n_c \kappa)\ \sigma'_f \tag{34}$$

or

$$\tau_f = (\kappa + \mu_e/n_c)\ \sigma'_e \tag{35-A}$$

and by use of Eq. 31

$$\tau_f = (\kappa + \mu_e/n_c)\ c_z\ \exp(-Be_f) \tag{35-B}$$

A comparison of Eqs. 24 and 34 yields

$$\mu_s = \mu_e + n_c \kappa \tag{36}$$

These equations show that the shear strength of a normally consolidated clay can be expressed as an explicit function of either the effective normal stress, or the equivalent consolidation pressure, or the water or void ratio

at failure, as also shown in Section 4. Eqs. 35 may be used for comparison of shear strengths of drained and undrained tests on normally consolidated clays when the pore-water pressures and effective stresses are unknown; see Section 7 and Fig. 32-A. It may be mentioned that σ'_e for normally consolidated clays is nearly identical with the values of the effective principal stress, σ'_1, at failure in direct shear tests and triaxial compression tests, BJERRUM (1954), but not in the case of triaxial extension tests; see Section 9 and Fig. 38.

The results obtained in tests on two batches of Vienna clay are summarized in Table 2.

Table 2

Strength Parameters for Vienna Clay

Batch	B	e_o	n_c	μ_s	ϕ'_s	μ_e	ϕ'_e	κ	c_z
I	8.3	0.85	1.72	0.504	26°45'	0.324	18°	0.105	122
V	8.4	0.84	1.70	0.485	25°50'	0.315	17°30'	0.100	116

The Effective Friction Component

The foregoing theoretical derivations and resulting failure conditions may primarily apply to clays which have been remolded at water contents close to the liquid limit and then reconsolidated to water contents between the liquid and plastic limits. The simple expressions for the effective cohesion component may not apply to clays remolded at relatively low water contents or to undisturbed clays in which some of the bonds between the particles have a character which is different from that in remolded clays. Under such conditions a method, proposed by TERZAGHI (1938), for separate determination of the effective friction and cohesion parameters may be used to advantage, and this method also excellently illustrates the physical meaning of the components.

The above-mentioned method is illustrated in Fig. 23. The basic assumptions are that the effective cohesion component is constant when the water content is constant, and that the rheological component also is constant when the water content and the rate of deformation or test duration are constant. Furthermore, it must also be assumed that there is no significant difference in the geometric structure of the test specimens in a given series at the time of failure. The points D, E, F on the stress-water content diagram in Fig. 23-B

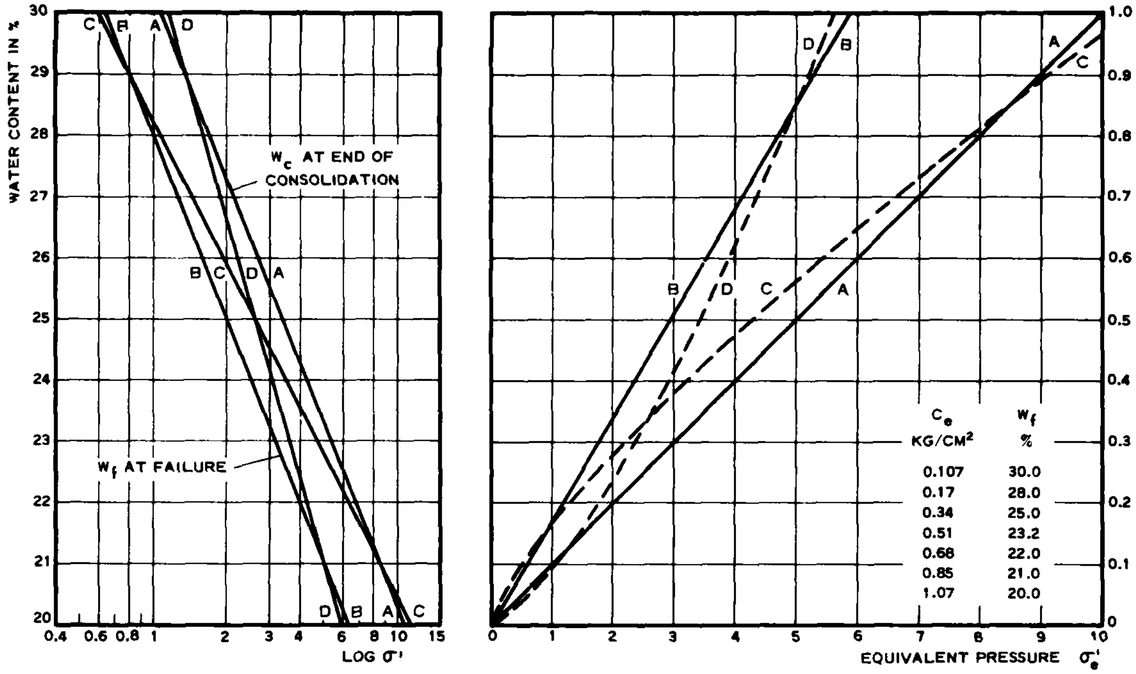
have the same water content at failure. The corresponding points D, E, F on the stress-strength diagram are determined as shown in Fig. 23-A. These points should and usually do lie on a straight line; the angle of inclination of this line, ϕ'_e , is defined as the effective angle of internal friction. The ordinate of the intercept G is the effective cohesion component, c_e , for the water content w_f of points D, E, F. This operation is repeated for other water contents, whereby data are obtained for establishing the relation between the effective cohesion component and water content. The graphical determination of ϕ'_e may in practice be replaced by numerical computations, using the coordinates of points D, E, F in the stress-strength diagram.

BJERRUM (1954), GIBSON (1953), and others have made many tests on remolded clays and determined the effective angles of internal friction by means of the Terzaghi method. In general, it was found that the angles obtained do not vary with the water content beyond normal scatter of test results. This establishes in part the reliability of the method, but for full confirmation of the physical interpretation of the results, it is necessary to compare the angles ϕ'_e with angles of inclination of failure planes in compression tests on the same clays. Such comparisons are discussed in Section 6, and it is shown that fairly satisfactory statistical confirmation is obtained for remolded clays. Theoretically it is also possible to determine the angle ϕ'_e by measuring the angles of inclination, α , of the failure planes and using the theoretical relation $\phi'_e = 90 - 2\alpha$; however, the method is not reliable because of considerable scatter in the individual values of α and also because of the influence on unknown anisotropic properties of the test specimen.

The Effective Cohesion Component

After having obtained various corresponding values of the effective cohesion component and water content by the Terzaghi method, one may first investigate if this component can be expressed as a linear function of the equivalent consolidation pressure or as an exponential function of the void ratio or water content, as indicated in Eqs. 29 and 33. An example of such evaluations is shown in Figs. 25 and 26, using data obtained in tests on Vienna clay.

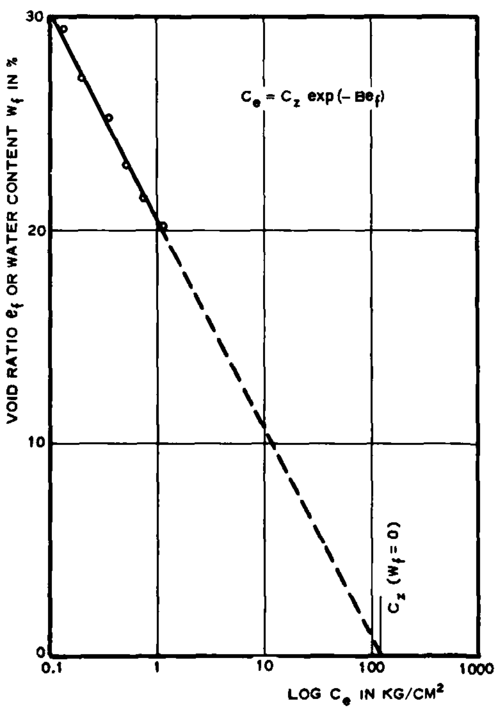
If the values of σ'_e are determined on basis of line AA in Fig. 25-A, corresponding to the virgin branch of the consolidation diagram, the relation between c_e and σ'_e is represented by the straight line AA in Fig. 25-B. If the values of σ'_e are determined by line BB, which is parallel to AA and



A - EQUIVALENT PRESSURE LINES

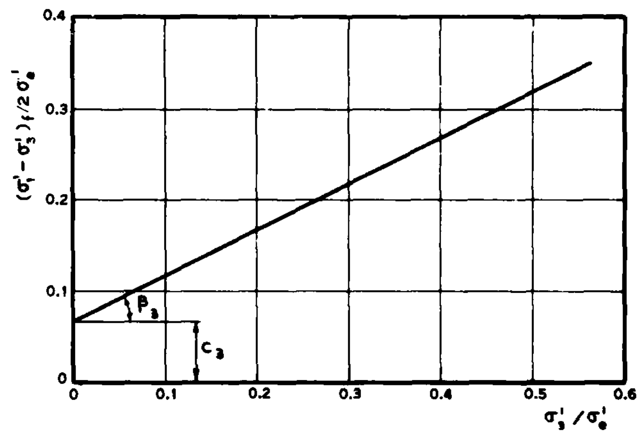
B - COHESION VERSUS EQUIVALENT PRESSURES

FIG. 25. COHESION AS A FUNCTION OF EQUIVALENT CONSOLIDATION PRESSURES



COHESION AS A FUNCTION OF WATER CONTENT OR VOID RATIO

FIG. 26



$$\frac{(\sigma'_1 - \sigma'_3)_f}{2\sigma'_e} = \frac{\cos \phi'_r}{1-\sin \phi'_r} \cdot \frac{c'_r}{\sigma'_e} + \frac{\sin \phi'_r}{1-\sin \phi'_r} \cdot \frac{\sigma'_3}{\sigma'_e}$$

$$\tan \beta_3 = \frac{\sin \phi'_r}{1-\sin \phi'_r} \qquad c_3 = \frac{\cos \phi'_r}{1-\sin \phi'_r} \cdot \frac{c'_r}{\sigma'_e}$$

$$\frac{(\sigma'_1 - \sigma'_3)_f}{2} = c_3 \cdot \sigma'_e + \sigma'_3 \tan \beta_3$$

AFTER BISHOP AND HENKEL (1957)

DETERMINATION OF FRICTION AND COHESION PARAMETERS FROM RESULTS OF TRIAXIAL TESTS

FIG. 27

represents the water contents at failure of normally consolidated test specimens, the line BB in Fig. 25-B is obtained. The tangents of the slopes of the two lines are equal to the coefficients κ. The values of κ and of σ_e' are different for the two lines, but the product $c_e = \kappa\sigma_e'$ is constant for a given water content. On the other hand, when values of σ_e' are determined by means of the line CC, which has a flatter slope than AA, the dashed line CC in Fig. 25-B is obtained. Unless test values of c_e corresponding to low values of σ_e' are available, the line CC gives the impression of being fairly straight and intercepting the ordinate axis. Similarly, the use of line DD, which has a steeper slope than AA, for determination of σ_e' results in the dashed line DD, which gives the impression of intercepting the abscissa axis.

In case there is doubt about the proper slope of the reference line for determination of σ_e', it is better to evaluate the test results by plotting $\log c_e$ versus w_f as shown in Fig. 26. When the points lie on a straight line, the effective cohesion component can be expressed by

$$c_e = c_z \exp(-Be_f) \tag{37-A}$$

where c_z is the intercept with the abscissa axis and B can be computed from the slope of the line. As previously explained, the coefficient c_z represents the theoretical value of the effective cohesion at zero void ratio, and it is a more significant soil parameter than κ, the value of which depends on the choice of a reference line for determination of values of σ_e'.

Extensive investigations of the shear strength of remolded cohesive soils have been made by BJERRUM (1954), who verified that the cohesion component of clays remolded at high water contents is proportional to σ_e' but found that this component for clays remolded at low water contents is expressed by

$$c_e = c_o + \kappa\sigma_e' \tag{37-B}$$

It is quite possible that this relation is correct since the shear strength characteristics of clays remolded at water contents close to the plastic limit may be different from those of clays remolded at high water contents. It is also possible that the reference lines used by Bjerrum for determination of the values of σ_e' did not have the proper slope and produced (c_e, σ_e') diagrams similar to CC and DD in Fig. 25-B. This possibility is enhanced by

the use of clays remolded at different water contents in a single test series, and Bjerrum realized that this testing procedure makes it difficult to obtain a proper reference line for determination of σ'_e . Bjerrum also used the method shown in Fig. 26 for evaluation of some of the test results and found a slight curvature of the line $(w_f , \log c_e)$ at low water contents, which imposes a limit on the applicability of Eq. 37-A.

Determination of Strength Parameters by Triaxial Tests

The proposed expressions for the shear strength in Eqs. 29 and 33 are based on results obtained in direct box or torsion shear tests. SKEMPTON and BISHOP (1954) have developed two methods for determination of the effective friction and cohesion parameters by means of triaxial compression tests; these methods are also described by BISHOP and HENKEL (1957). In the first or direct method a diagram similar to that in Fig. 23 is used, and Mohr circles are drawn for different stress conditions but at constant water content. The envelope for such a set of circles is identical with the shear strength line for the particular water content or void ratio; hence, $\phi'_m = \phi'_r = \phi'_e$. The intercept of the envelope, c'_r , represents the effective cohesion component for the particular void ratio. Envelopes for various void ratios yield the same values of ϕ'_r but different values of c'_r . The second and very elegant method is illustrated in Fig. 27. The basic triaxial failure condition, Eq. 9-A, is transformed into that shown in Eq. 9-D. Division by σ'_e , the equivalent consolidation pressure obtained from the virgin branch of a triaxial consolidation test under all-round pressure, gives this equation the dimensionless form

$$\frac{(\sigma'_1 - \sigma'_3)_f}{2\,\sigma'_e} = \frac{c'_r}{\sigma'_e}\,\frac{\cos\phi'_r}{(1 - \sin\phi'_r)} + \frac{\sigma'_3}{\sigma'_e}\,\frac{\sin\phi'_r}{(1 - \sin\phi'_r)} \qquad (38\text{-A})$$

which is similar to that of Eq. 30. By plotting $(\sigma'_1 - \sigma'_r)_f/2\sigma'_e$ versus σ'_3/σ'_e , a straight line is obtained with the angle of inclination β_3 , the intercept c_3 , and the equation

$$\frac{(\sigma'_1 - \sigma'_3)_f}{2\,\sigma'_e} = c_3 + \frac{\sigma'_3}{\sigma'_e}\tan\beta_3 \qquad (38\text{-B})$$

which also may be written

$$\tfrac{1}{2}(\sigma'_1 - \sigma'_3)_f = c_3\sigma'_e + \sigma'_3\tan\beta_3 \qquad (38\text{-C})$$

Values ϕ'_r and c'_r are obtained by equating the coefficients in Eqs. 38-A and 38-B, as shown in Fig. 27, or

$$\sin \phi'_r = \frac{\tan \beta_3}{1 + \tan \beta_3} \quad \text{and} \quad c'_r = \sigma'_e \frac{c_3(1 - \sin \phi'_r)}{\cos \phi'_r} = \sigma'_e \kappa_t \qquad (38\text{-D})$$

The parameters ϕ'_r and κ_t correspond to ϕ'_e and κ in the equivalent method suggested for evaluation of the results of direct shear tests, Eq. 30. The method has the advantage that it facilitates evaluation of the test results and determination of the proper average values of the parameters from scattered test data, but it should be used only when significant data for determination of σ'_e are available and when the effective cohesion component can be expressed as a constant times σ'_e .

Similar equations can also be obtained by plotting $\frac{1}{2}(\sigma'_1 - c'_3)/\sigma'_e$ versus $\frac{1}{2}(\sigma'_1 + \sigma'_3)/\sigma'_e$ which form a straight line with the equation

$$\frac{1}{2}(\sigma'_1 - \sigma'_3) = c_s \sigma'_e + \frac{1}{2}(\sigma'_1 + \sigma'_3) \tan \beta_s \qquad (38\text{-E})$$

and the values of ϕ'_r and c'_r can then be determined by equations which are equivalent to Eq. 9-C.

Values of ϕ'_r and κ_t , obtained in triaxial tests at the University of London, are summarized by HENKEL (1958). It may be noted that values of κ_t quoted by Henkel are considerably smaller than corresponding values of κ obtained in direct shear tests, Fig. 24, GIBSON (1953), BJERRUM (1954). This difference in results of direct shear tests and triaxial tests may in part be caused by various sources of error in the two types of tests, but it may also in part be due to the fact that the intermediate principal stress at failure in direct shear tests has a value between the major and minor principal stresses, which may cause the strength to be slightly higher than that obtained in triaxial compression or extension tests; see Section 9 and Fig. 40.

Comments on Validity and Applications

When the results of strength tests are plotted as shown in Figs. 24 and 27 and form a straight line, Eqs. 29, 33, or 38 constitute mathematically correct expressions for the shear strength in the failure zones irrespective of the value of the strength components or parameters in other directions. This statement applies also to test results expressed by the Coulomb condition of failure. The problem of possible variations of the strength components or

parameters in other directions than that of the planes of failure is discussed in Section 6.

The effective cohesion component for a given clay is expressed as a function of the void ratio, Eqs. 33 and 37-A, but it is undoubtedly also a function of the geometric structure of the clay. However, the influence of minor initial variations in structure on the failure conditions will probably be relatively small when the deformations tend to produce similar structures at failure. These expressions for the effective cohesion component are based on the results of tests on remolded clays with a normal range of water contents or void ratios, and it is possible that the parameters or the form of the relation may change when the void ratios become very small or very large. Modification of the equations may also be required for undisturbed clays in which stronger chemical bonds or cementation between the clay particles have been formed in course of time.

The strengths obtained by standard tests, and expressed by Eqs. 29, 33, or 38, are larger than the actual long-term strengths because the test data and the equations include a time-dependent or rheological component, but so do the same test data expressed by the Coulomb or Mohr-Coulomb failure conditions. Methods for determination of the rheological component and available data on the time-dependent decrease in strengths are discussed in Section 7.

The parameters in the Coulomb and Mohr-Coulomb failure conditions vary with the stress history of a clay, and a graphical representation of these failure conditions consists of a family of shear strength lines and hysteresis loops. The same test data may be expressed by Eqs. 29, 33, and 38, in which the parameters are independent of the stress history of the clay. Furthermore, a graphical representation is reduced to a single line as shown in Figs. 24 and 27. This constitutes a considerable simplification from a theoretical point of view, and it furnishes a simple and consistent explanation of the variations in shear strength. However, application of the failure condition expressed by these equations and diagrams requires knowledge not only of the pore-water pressures and effective stresses but also of the void ratios at the moment of failure. Advances have been made in developing methods for estimating changes in pore-water pressures and void ratios under laboratory testing conditions, but these methods have not yet been verified for stress conditions encountered in many practical problems. Therefore, the proposed methods for determination of the effective friction and cohesion components may currently and primarily be of interest in research and for estimating

changes in the parameters and establishing proper limits for the use of the Coulomb and Mohr-Coulomb failure criteria in the analysis of practical problems.

6. PLANES OF FAILURE AND INFLUENCE OF ANISOTROPY

Tests on Vienna Clay and Little Belt Clay

In the previously discussed direct shear tests the orientation of the flaky clay particles was horizontal or parallel to the induced planes of failure. Several series of unconfined compression tests were performed in order to obtain an indication of the influence of the direction of orientation of the clay particles on the compressive strength, q, and the angle of inclination of the planes of failure, α, and also to determine whether the corresponding values of $\phi_\alpha = 90 - 2\alpha$ agree with ϕ'_e or ϕ'_s.

Remolded Vienna and Little Belt clays were reconsolidated in the shear boxes under a normal pressure of 5.0 kg/cm^2. Three types of test specimens were cut from the reconsolidated clays in such a manner that the angle between the axis of the test specimen and the planes of orientation of the clay particles was 90° for type I, 45° for type II, and 0° for type III specimens; see Fig. 28. The test specimens had a length of 4 cm and a square cross section with a side length of 2 cm. The square cross section was chosen in order to facilitate preparation of the specimens and measurement of the inclination of the failure planes. Some drying was allowed to take place during the preparation of the specimens, and the final water content corresponded to an equivalent consolidation pressure of about 6 kg/cm^2. The specimens were stored in a closed vessel for several days in order to increase the uniformity of distribution of the water content. The compression tests were performed with load increments of about 2.5 per cent of the estimated failure load and a time interval of 2 minutes between load increments. The inclination of the failure planes was measured upon completion of the test, whereupon the specimen was weighed and dried and the water content computed.

Four specimens of each type were prepared and tested, and good agreement of the test results was generally obtained, but there were occasional deviations. It was assumed that major deviations primarily were caused by irregularities within the test specimens or in preparation of the end surfaces; therefore, each figure in the summary of the test results, shown in Fig. 28, represents the average result of the three tests in best agreement out of four tests. The photographs in Fig. 28 show test specimens which have been distorted by drying, since the shrinkage was greatest in a direction perpendicular to the planes of orientation of the clay particles.

Deformations. Appreciable deformations after failure were in some cases

permitted in order to obtain full development of the planes of failure. Type I test specimens were subjected to considerable bulging and lateral deformations after failure, which caused a slight rotation of the planes of failure. On the other hand, deformations after failure of types I and III specimens consisted primarily of sliding along the planes of failure. Type II specimens of Little Belt clay were subjected to plastic flow in a direction parallel to the planes of orientation of the clay particles, in addition to sliding along the planes of failure. Type III specimens of Vienna clay had a tendency to vertical splitting, which in some cases was more pronounced than shown in Fig. 28. A reorientation of the clay particles in the zones of failure was not observed but may occur, especially in slow tests.

Compressive strengths. The compressive strengths of the three types of test specimens cannot be compared directly because of slight variations in water content and strength of the individual specimens. However, TERZAGHI and JANICZEK (1931) have shown that the unconfined compressive strength of a normally consolidated clay can be expressed as a linear function of the equivalent consolidation pressure, and this was verified in the experiments by the writer, HVORSLEV (1937, 1938). Therefore, the ratio q/σ'_e can be used for comparison of the strengths of the three types of specimens.

As shown in Fig. 28, type II specimens have compressive strengths between those obtained for types I and III. Type 1 specimens of Vienna clay have 14 per cent greater strength than those of type III, but type III specimens of Little Belt clay have 20 per cent greater strength than type I specimens, and this difference in behavior prevents formulation of general conclusions. The results obtained for Vienna clay, batch IX, are verified by previous incomplete test series on other batches, but lack of material prevented performance of additional unconfined compression tests on Little Belt clay. The variations in compressive strength may be caused in part by differences in cohesion and friction parameters and in part by differences in pore-water pressures developed during tests on the three types of specimens. A few compression tests were also made on type I specimens of Vienna clay with circular cross section, and it was found that they had 8 per cent greater strength than those with a square cross section.

Inclination of failure planes. Average measured angles of inclination, α , and the corresponding values of $\phi_\alpha = 90 - 2\alpha$, which are not considered fully reliable are shown in parentheses in the table of Fig. 28. For type I specimens the values of α were probably increased slightly by deformations

after failure, and there was considerable scatter in the measured values of α for type III specimens of Little Belt clay. The test results show excellent agreement between values of ϕ_α, corresponding to the primary planes of failure in type II specimens, and the angles of effective internal friction, ϕ'_e, obtained in the direct shear tests. As previously mentioned, the duration of the direct shear tests on Little Belt clay was probably too short, and it is not known to what extent the errors affecting $\phi'_e = 10^\circ$ compensate each other.

Many unconfined compression tests were performed on types I and III specimens of other batches of Vienna clay. The axial deformations at the end of these tests were smaller than for the tests mentioned above, and the measured angles of inclination of the failure planes may be more reliable. The following average values of ϕ_α were obtained in these tests:

Vienna clay, batch		III	V	VI
Specimens, type I,	$\phi_\alpha =$	18°	19°	19°
Specimens, type III,	$\phi_\alpha =$	19°	19°	19.5°

The corresponding values of ϕ'_e were $17^\circ 30'$ to 18° and those of the angle of shear strength, ϕ'_s, were $25^\circ 50'$ to $26^\circ 45'$. It is seen that the values of ϕ_α for types I and III specimens are slightly larger than ϕ'_e but much smaller than ϕ'_s.

Tests on Other Clays

The results of compression tests on various remolded English clays, performed at the University of London, are summarized by GIBSON (1953). It was found that the failure planes rotate and that the value of the angle of inclination, α, increases with increasing strain. Values of ϕ_α, corresponding to the initial values of α, were in excellent agreement with values of ϕ'_e which had been corrected for the influence of surface energy or dilatation but were 1° to 2° larger than uncorrected values of ϕ'_e.

BJERRUM (1953) reports that values of ϕ_α and ϕ'_e, obtained in tests on five silty to very plastic Swiss clays, were in good agreement, the deviations being of the order of $\pm 1.5^\circ$. BJERRUM (1954) also performed two extensive series of compression tests on remolded talus clay from Zurich. In one series the compression tests were performed immediately after remolding and in the other series after hydrostatic reconsolidation. The average values of ϕ_α obtained in the two series were in good agreement in spite of differences in water content and strength, but the deviations of individual measurements were

I II III

VIENNA CLAY - BATCH IX

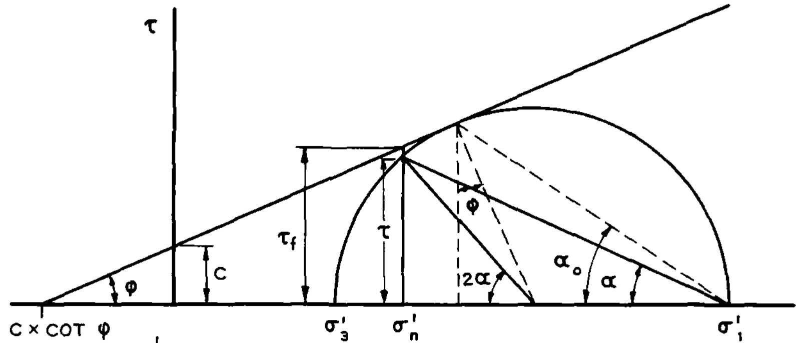
I II III

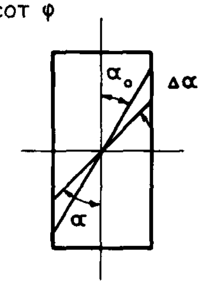
LITTLE BELT CLAY - BATCH III

	VIENNA CLAY IX			LITTLE BELT CLAY III		
	I	II	III	I	II	III
q/σ'_e	0.48	0.44	0.42	0.38	0.41	0.46
RELATIVE STRENGTH	109	100	95	93	100	112
α	(39°)	36°30'	36°	(43°)	40°15'	(42°)
$\varphi_\alpha = 90 - 2\alpha$	(12°)	17°	18°	(4°)	9°30'	(6°)
φ'_e	(BATCH V)	17°30'		(BATCH I)	(10°)	

INFLUENCE OF PARTICLE ORIENTATION IN COMPRESSION TESTS

FIG. 28.





$$\tau = \frac{1}{2}(\sigma'_1 - \sigma'_3) \ \text{SIN } 2\alpha$$

$$\sigma'_n = \frac{1}{2}(\sigma'_1 + \sigma'_3) - \frac{1}{2}(\sigma'_1 - \sigma'_3) \ \text{COS } 2\alpha$$

$$\tau_f = C + \sigma'_n \ \text{TAN } \varphi$$

$$\frac{\tau_f}{\tau} = \frac{1 - \text{SIN } \varphi \ \text{COS } 2\alpha}{\text{COS } \varphi \ \text{SIN } 2\alpha}$$

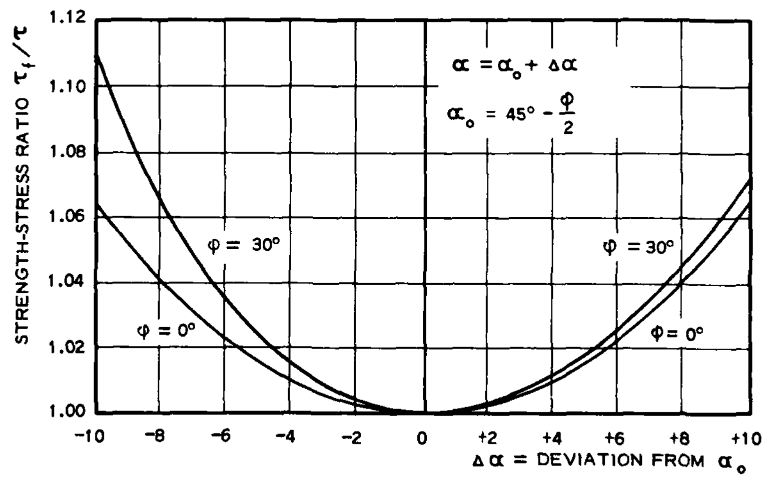


FIG. 29. STRENGTH-STRESS RATIO VERSUS INCLINATION

rather large, the standard deviation of ϕ_α being 8° to 9°. It was concluded that ϕ_α can be determined by tests on remolded clays with or without reconsolidation, but single measurements are not reliable, and the number of tests performed must be large enough to determine a statistical average value.

CASAGRANDE and WILSON (1953-B, p. 36) state that measured angles of inclination of failure planes and the corresponding values of ϕ_α agree well with the angles of shear strength, ϕ'_s, obtained in triaxial tests at Harvard University on silty to highly plastic clays having values of ϕ'_s ranging from 20° to 34°. Detailed test data in support of the statement are not presented in this report, but CASAGRANDE and RIVARD (1959) state that values of ϕ_α for an undisturbed plastic clay varied from 12° to 30°, and that the average was 21°, or 23° when maximum and minimum values were excluded. The corresponding values of ϕ'_s ranged from 21° to 24°. On the other hand, HABIB (1953) found $\alpha = 45^\circ$ or $\phi_\alpha = 0$ for three clays and refers to similar results obtained by TSCHEBOTARIOFF and BAYLISS (1948).

Theoretical Considerations

CASAGRANDE and CARRILLO (1944) have developed analytical and graphical methods for determination of the angle of inclination of and the strength in the failure planes of anisotropic, cohesionless or purely cohesive soils. HANK and McCARTHY (1948), using modified Mohr stress circles, have extended these methods to soils possessing both cohesion and internal friction. These theories show that anisotropy may have a marked influence on the optimum angle of inclination of the failure planes. The simplest case is that of a purely cohesive soil in which the directions of the maximum and minimum values of the cohesion, c_1 and c_2, coincide with those of the principal stresses. Assuming that the variation of the cohesion is represented by an ellipse, the optimum angle of inclination, α_o, between the direction of the major principal stress and the failure planes, and the shear strength, τ_f, in these planes are expressed by

$$\tan \alpha_o = \sqrt{c_1/c_2} \qquad \tau_f = 2c_1c_2/(c_1 + c_2) \tag{39}$$

Reference is made to the above-mentioned papers for the more complicated cases of soil possessing internal friction.

An indication of the influence of anisotropy or other irregularities can also be obtained by determining the strength-stress ratio, τ_f/τ, for various

planes in an isotropic test specimen having constant values of c and ϕ in the Coulomb failure criterion. As shown in Fig. 29, the strength-stress ratio is expressed by

$$\frac{\tau_f}{\tau} = \frac{1 - \sin\phi \cos 2\alpha}{\cos\phi \sin 2\alpha} \tag{40}$$

The ratio is equal to unity for $\alpha = \alpha_o = 45 - 1/2\phi$. Values of τ_f/τ for deviations, $\Delta\alpha$, from the optimum inclination, or $\alpha = \alpha_o + \Delta\alpha$, have been computed for $\phi = 0^o$ and $\phi = 30^o$ and are shown graphically in Fig. 29. A standard deviation of 8^o in ϕ_α , as found by BJERRUM (1954), corresponds to $\Delta\alpha = \pm 4^o$ and $\tau_f/\tau = 1.01$ to 1.015. This means that the failure may occur in a plane with the inclination $\alpha = \alpha_o \pm 4^o$ when the shear strength in this plane is 1.0 to 1.5 per cent smaller than in a plane with the inclination α_o . Such a small difference in strength might be caused by anisotropy, irregularities within the test specimen, or nonuniform distribution of the external stresses on the end surfaces of the test specimen.

Difference in friction and cohesion parameters for the test specimens shown in Fig. 28 cannot be definitely determined by means of the above-mentioned theories, since the influence of anisotropy on changes in pore-water pressures is not considered in these theories and was not determined during the tests.

Summary and Comments

Theoretical considerations show that anisotropy and minor irregularities in strength properties and stress distribution may cause considerable variations of the angle of inclination, α , of the failure planes, and measured values of this angle are subject to an appreciable amount of scatter. However, average values of $\phi_\alpha = (90 - 2\alpha)$ for remolded clays--obtained by the writer, Gibson, and Bjerrum--agree well with or are slightly larger than ϕ'_e , but are much smaller than ϕ'_s . These results do not furnish a definite answer to the hypothetical question of whether the effective cohesion component, c_e , is the same in all directions for an isotropic clay, but they do indicate that the proposed failure criterion and corresponding values of ϕ'_e are in better agreement with the test data than are values of ϕ'_s obtained by standard methods of evaluation and the Coulomb failure criterion. Published data on the inclination of failure planes in tests on undisturbed clays are limited in extent and of contradictory character.

Data in Fig. 28 on the influence of particle orientation on the unconfined compressive strength are of preliminary character and do not permit formulation of definite conclusions, since the directions of particle orientation giving maximum and minimum strengths are different for Vienna clay and Little Belt clay. However, the difference between maximum and minimum strengths is appreciable, 14 to 20 per cent, for both clays and is too large to be neglected, at least in research. Much more detailed investigations of the influence of orientation of clay particles and of anisotropy in general are needed. Triaxial tests should be performed on specimens with horizontal, inclined, vertical, and random orientation of the clay particles. The deformation characteristics, inclination of failure planes, pore-water pressures, strengths, and the effective cohesion and friction parameters should be determined for each type of orientation of the clay particles.

7. THE RHEOLOGICAL COMPONENT AND PROPERTIES

Early Investigations of Plastic Deformations

TERZAGHI (1931-A) reviewed the concepts and discussed the physical mechanics of plastic flow of clays. He pointed out that the plastic deformations may start at a shear stress considerably below the shear strength determined by laboratory tests and may continue for long periods of time, and he emphasized the importance of these phenomena in the design of foundation structures. These considerations were primarily based on the results of undrained compression tests on plastic clays.

Prompted by the above-mentioned paper by Terzaghi, the writer investigated plastic deformations of Little Belt clay under fully drained conditions. The test was performed with the torsion shear apparatus, Fig. 11, and the test specimen was strongly overconsolidated in order to avoid a gradual decrease in void ratio and corresponding increase in cohesion with increasing shear stresses, which might impede the full development of plastic deformations. The shear load increment of 0.1 kg/cm^2, and the time interval between load increments was about ten days. Total horizontal and vertical displacements for each load increment are shown in Fig. 30-A. Failure occurred 10 to 15 minutes after the last load increase, and the actual shear strength is undoubtedly slightly less than that corresponding to the final shear load, 0.50 kg/cm^2. The relation between the shear strain, γ, and the twist, θ, of the torsion shear apparatus is

$$\gamma = \theta \frac{2}{3} \frac{R_2}{H_e} \frac{1 - n^3}{1 - n^2} = 4.64\ \theta \text{ radians} \tag{41}$$

where $n = R_1/R_2$, R_1 and R_2 are the inside and outside radii, and H_e is the effective height of the test specimen. The actual height was 1.45 cm, but the shear strains are not uniformly distributed, and other tests indicated that the equivalent height of a test specimen with uniform shear strains is about 1.0 cm; HVORSLEV (1937).

Only the total deformations were observed. However, elastic deformations took place within a few hours after load application, and subsequent deformations were plastic or permanent. Plastic deformations for each load increment and in the period from 100 to 240 hours after each load application are shown in Figs. 30-B and -C. The inclination of tangents to the curves in Fig. 30-B represents the velocity of plastic deformations. These velocities were

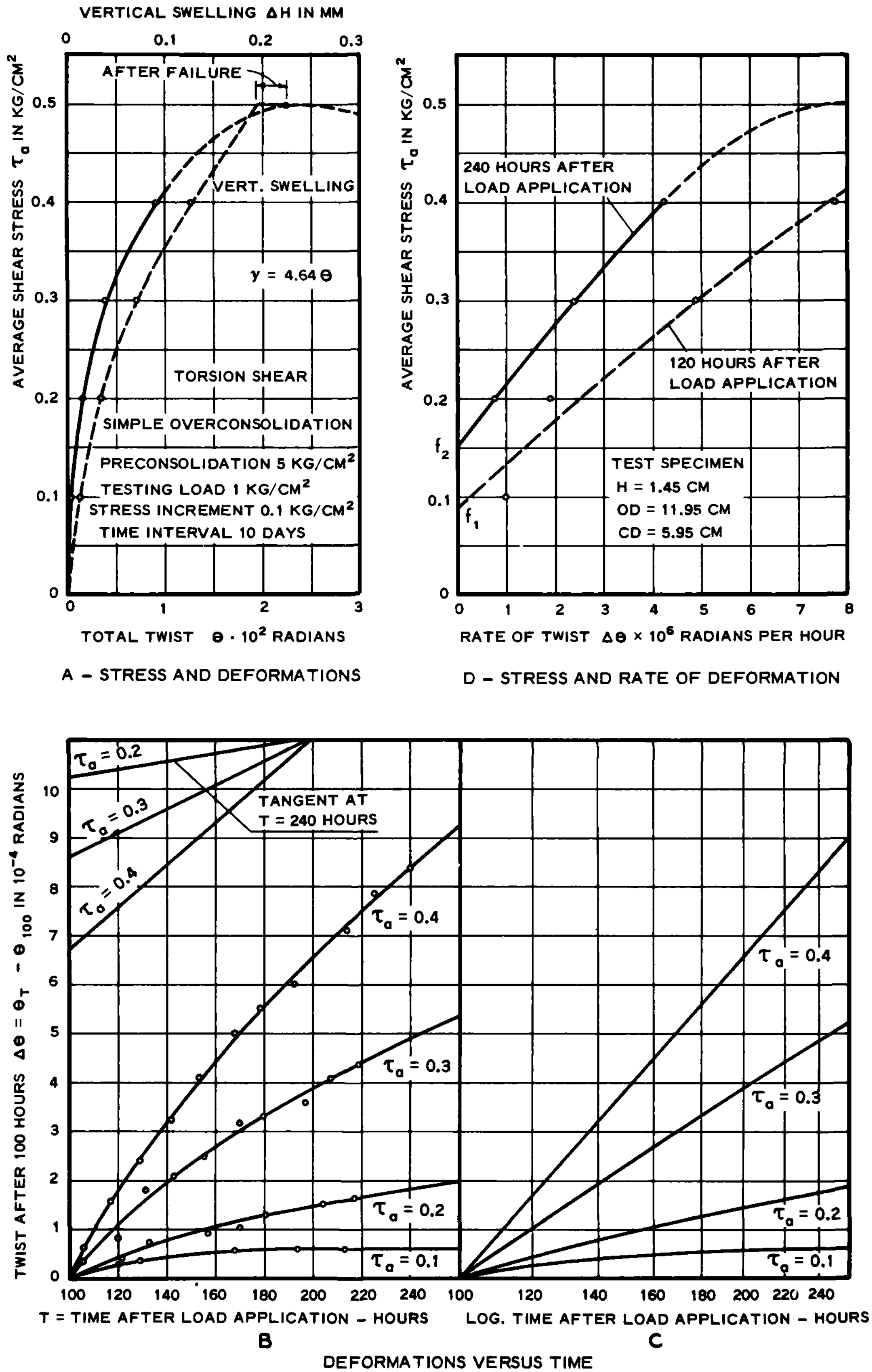


FIG. 30. PLASTIC DEFORMATIONS OF LITTLE BELT CLAY BEFORE FAILURE

determined for 120 and 240 hours after each load application, and are shown in Fig. 30-D. It is seen that these velocities were zero at 120 hours for shear stresses less than 0.09 kg/cm^2, and at 240 hours for shear stresses less than 0.15 kg/cm^2 or about 30 per cent of the shear strength. At greater shear stresses the velocities at 120 and 240 hours after load application increased approximately linearly with the increase in shear stress until failure was approached. Similar relations would undoubtedly have been obtained for still longer periods after load application, had the time intervals been increased.

The diagrams in Fig. 30-C show that the plastic deformations for a considerable period increased linearly with the logarithm of time. However, the deformations for the stress $\tau_a = 0.1$ kg/cm^2 ceased after a period of 220 hours. At the same time after load application the diagrams for $\tau_a = 0.2$ and 0.3 kg/cm^2 exhibit a slight curvature, and it is possible that the deformations for these stresses would also cease in the course of time. The diagram for $\tau_a = 0.4$ kg/cm^2 is straight for the entire time interval of 10 days, but this period was too short for estimating whether the deformations at this stress would continue at a decreasing rate and ultimately cease or cause failure of the test specimen.

As shown in Fig. 30-A, swelling occurred after each load increment and also after failure. This means that an increase in shearing stresses caused a decrease in pore-water pressure, but this increase was equalized in the course of time with a corresponding increase in void ratio and decrease of the friction and cohesion components of strength. This primary decrease in strength with time contributed to the elastic and plastic deformations. However, for the test described, the primary swelling is theoretically 99 per cent complete in less than 100 hours, and plastic deformations occurring more than 100 hours after load application may represent the influence of the viscous or rheological strength component, secondary swelling, and thixotropic and structural changes in strength.

Recent Rheological Investigations

Over many years HAEFELI (1939, 1953) made extensive field and laboratory investigations of continuing plastic deformations of snow and ice, extended these investigations to soil, and called attention to the great importance of such deformations in the design of foundation structures. HAEFELI and SCHAERER (1946) also suggested methods for determination of shear-strain relations for soils by means of triaxial tests.

GEUZE (1948, 1960) investigated plastic deformations of cohesive soils in the field and also in the laboratory by means of drained direct shear tests. He found a linear relation between the shear strains and the logarithm of time in both field and laboratory investigations. This relation can be expressed by

$$\gamma_t = \Delta\tau \ (\gamma_d + \ddot{\gamma} \log t) \tag{42}$$

where γ_t is the total strain at the time t , γ_d is the strain one day after placing the stress increment $\Delta\tau$, t is the time in days after placing the stress increment, and $\ddot{\gamma}$ is the increase in strain for an increase of t corresponding to a full logarithmic cycle or a tenfold increase of t . Both γ_d and $\ddot{\gamma}$ refer to $\Delta\tau = 1$ and depend also on the total stress, τ . The rate of strain is obtained by differentiation of Eq. 42,

$$\frac{d\gamma_t}{dt} = \Delta\tau \ \ddot{\gamma} \ \frac{2.3}{t} \tag{43}$$

The elastic strains attain their maximum value in less than one day, and subsequent increases in strain are plastic strains. Therefore, Eq. 43 represents the rate of plastic strain and shows that this rate varies inversely with time. Eqs. 42 and 43 apply also, with minor modifications and limitations, to the test data shown in Fig. 30. Geuze also suggested that the volumetric strains should be considered in computing the pure shear strains, and he called attention to the similarity between the plastic shear strains and the volumetric strains caused by secondary consolidation.

GEUZE and TAN (1953) and GEUZE (1960) investigated the shear deformations of clays at constant water content by means of torsion shear and triaxial tests. The tests were performed and evaluated in accordance with the principles and theories of rheology and plasticity, and the authors deserve credit for introducing these principles into the field of soil mechanics. The mean compressive stress was held constant in order to produce states of pure shear, and the elastic and plastic deformations were separated by unloading the test specimen before each load increase and observing the rebound. It was found that the shear strains were reversible or elastic in character up to $\tau = f_o$, called the yield limit. For stresses greater than the yield limit, part of the strains were permanent or plastic. For the soils investigated it appears that these plastic strains approach a constant velocity

during the rather limited time intervals between the load increments, and that this terminal velocity increases linearly with the stress increase, as expressed by

$$(\tau - f_o) = \eta \frac{d\gamma}{dt} \tag{44}$$

where η is the coefficient of structural viscosity, also called the Bingham viscosity since the behavior of such clays can be represented by a Bingham rheological model. However, the Bingham concept is being disputed by some investigators, and the yield limit f_o is a theoretical rather than a definite physical limit since there is a transition between the state of pure elastic deformations and that of plastic deformations with velocities given by Eq. 44. When the shear stress exceeds another limiting value, called the flow limit and designated by f_f, the rate of plastic strains becomes greater than that indicated by Eq. 44, and the authors suggest that this greater rate of flow ultimately may increase with time and cause failure of the soil. In support of this suggestion, reference is made to experiments by CASAGRANDE and WILSON (1951). GEUZE (1960) proposes that the design of earth and foundation structures should be based on the above-mentioned yield and flow limits rather than on the shear strengths obtained in currently used laboratory and field tests.

Determination of the elastic and plastic components of the total strains by complete unloading of the test specimen before each load increase may cause changes in the stress-strain properties of the soil, which in many cases may be objectionable. However, unloading may be avoided by utilizing the suggestion by Wilson that the elastic strains are approximately equal to the instantaneous strains, which can be determined as the zero time intercepts of the stress-strain curves for individual load increments; see CASAGRANDE and WILSON (1949) and WILSON and DIETRICH (1960).

Eqs. 43 and 44 represent two basic types of plastic deformations of clays. It may in many cases require very long time intervals between load increments in order to determine which of these types of flow actually occurs, as is readily seen by examination of Figs. 30-B and 30-C. GOLDSTEIN (1957, 1958) has developed more complete equations for the plastic deformations, which take the effect of previous load increments into consideration. The above-mentioned relatively simple relations may be altered when the clays undergo thixotropic or structural changes in strength, or when migration of

pore water occurs during the plastic deformations. The deformations of clays may in many cases be represented by rheological models for which mathematical expressions of the relations between stresses, strains, and time have been developed and are described in various papers and books on rheology, see for example REINER (1954). Applications of visco-elastic stress-strain theories and rheological models to specific soil problems and for various assumed relations are explained in the excellent paper by SCHIFFMAN (1959).

The Rheological Component of Strength

A comparison of the results of slow and quick direct shear tests on Vienna clay and Little Belt clay, Fig. 31, shows that quick tests yield lower strengths than slow drained tests for normally consolidated and slightly overconsolidated clays, but higher strengths for strongly overconsolidated clays. The primary causes of these differences in strength are (1) the development of positive or negative pore-water pressures during quick tests and corresponding decreases or increases in void ratio during slow drained tests, and (2) changes in the viscous or rheological component of strength with time or the rate of deformation.

The part of the difference in shear strengths obtained by quick and slow direct shear tests, attributable to the rheological component, can be determined for normally consolidated clays by means of Eq. 35 and for overconsolidated clays by a direct comparison of strengths at the critical degree of overconsolidation, when there are no changes in pore-water pressure or void ratio during a shear test. The data shown in Fig. 31 cannot be used for the abovementioned purpose because the quick and slow tests were made with different batches of clays, but comparable data were obtained in tests on batch I of Vienna clay.

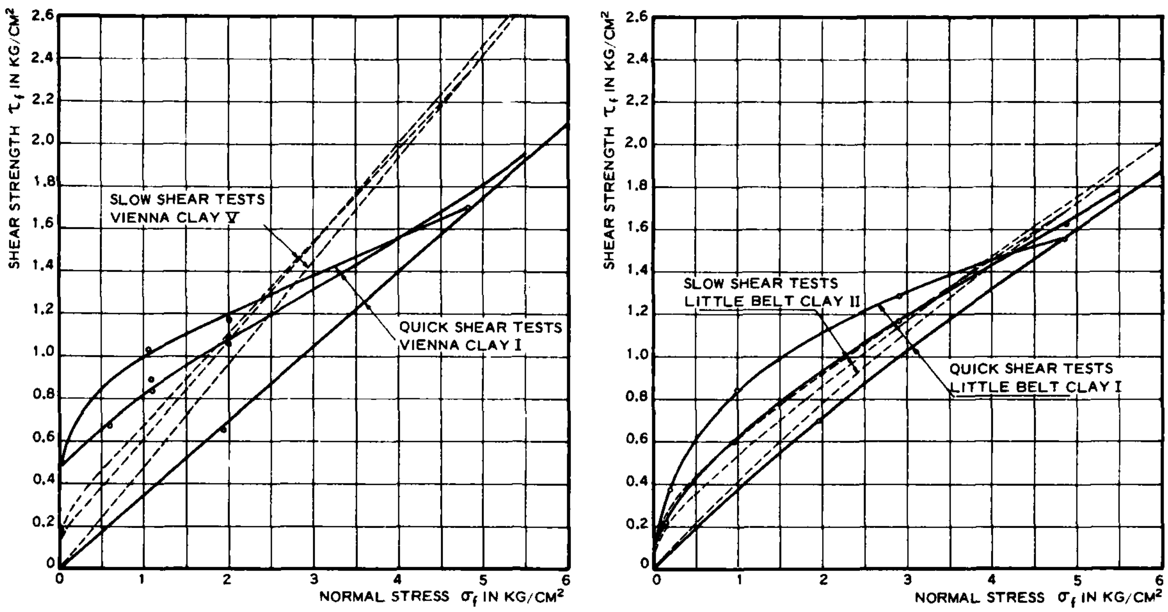
The strength parameters for Vienna clay I are shown in Fig. 32-A, and the strength obtained in slow drained tests on normally consolidated samples is

$$\tau_f = 0.324\ \sigma'_f + 0.105\ \sigma'_e \qquad (45\text{-A})$$

or by use of Eq. 35 and $n_c = 1.72$

$$\tau_f = 0.189\ \sigma'_e + 0.105\ \sigma'_e = 0.294\ \sigma'_e \qquad (45\text{-B})$$

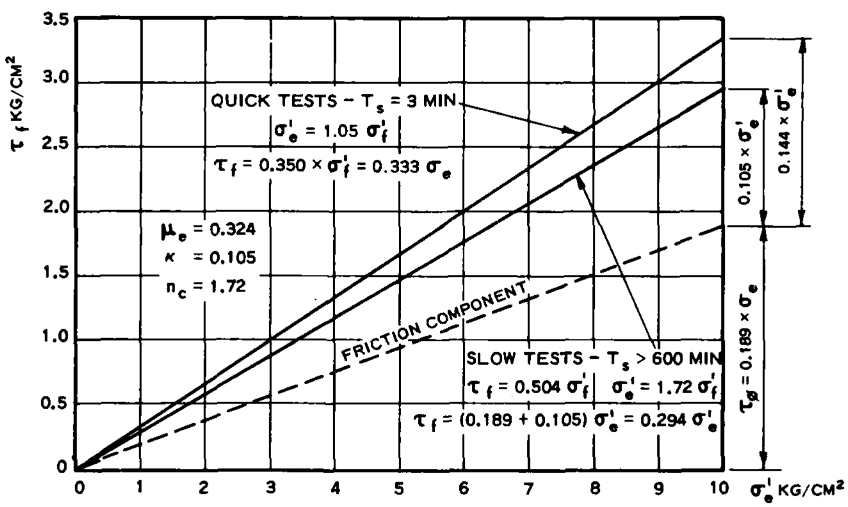
where $\tau_\phi = 0.324\ \sigma'_f = 0.189\ \sigma'_e$ is the friction component and $c_e = 0.105\ \sigma'_e$ is the effective cohesion component.



A – VIENNA CLAY

B – LITTLE BELT CLAY

FIG. 31. RESULTS OF QUICK DIRECT SHEAR TESTS



A – NORMAL CONSOLIDATION

ZERO VOLUME CHANGE		τ_ϕ	SLOW TESTS T_s > 600 MIN		QUICK TESTS T_s = 3 MIN		RATIOS SLOW/QUICK	
$5-0-\sigma'_f$	σ'_f	$0.324\ \sigma'_f$	τ_f	c_e	τ_f	c_e	τ_f	c_e
$\Delta H = 0$	1.0	0.32	0.68	0.36	0.82	0.50	0.83	0.72
$\Delta \sigma_e = 0$	1.3	0.42	0.81	0.39	0.91	0.49	0.89	0.80

B – OVERCONSOLIDATION

FIG. 32. INFLUENCE OF RHEOLOGICAL COMPONENT FOR VIENNA CLAY I

During quick shear tests on normally consolidated samples the water content in the shear zone decreased 0.4 per cent, corresponding $\Delta_e = 0.011$, which inserted in Eq. 23 yields $n_c = 1.10$. However, the time required for dismantling the apparatus and cutting samples from the shear zone was about equal to the duration of a quick shear test, and it is probable that a part of the change in water content in the shear zone is caused by internal migration of water after failure. Therefore, it is assumed that the actual change in water content at the moment of failure only was 0.2 per cent, corresponding to $n_c = 1.05$ or $\sigma'_e = 1.05\ \sigma_f$, where σ_f is the total normal stress at failure and also the effective stress at the start of the test. The shear strength obtained in the quick test on normally consolidated samples can then be expressed by

$$\tau_f = 0.350\ \sigma_f = 0.333\ \sigma'_e \qquad (46\text{-A})$$

or by assuming that the friction component is not affected by time,

$$\tau_f = 0.189\ \sigma'_e + 0.144\ \sigma'_e \qquad (46\text{-B})$$

where $c_e = 0.144\ \sigma'_e$ is the effective cohesion component obtained in quick tests. A comparison of Eqs. 45 and 46 yields the following strength-time ratios,

$$\frac{\tau_f\ (\text{slow})}{\tau_f\ (\text{quick})} = \frac{0.294}{0.333} = 0.88 \quad \text{and} \quad \frac{c_e\ (\text{slow})}{c_e\ (\text{quick})} = \frac{0.105}{0.144} = 0.73$$

Data obtained by tests on overconsolidated Vienna clay I are summarized in Fig. 32-B. The critical degree of cyclic overconsolidation, at which there is no change in pore-water pressure or void ratio during a shear test, can be determined approximately by examination of Fig. 21. The surface movement, ΔH , is zero for $\sigma'_f = 1.0$, and the diagrams representing equivalent consolidation pressures at the start and end of the shear tests intersect each other at $\sigma'_f = 1.3$. Corresponding values of τ_f and c_e were then determined, assuming that the friction component $\tau_\phi = 0.324\ \sigma'_f$ for both slow and quick tests. The average values of the strength-time ratios are 0.86 for τ_f and 0.76 for c_e .

The evaluation of these test results is based on various approximations, and does not consider a possible but minor influence of differences in

thixotropic changes in strength or differences in remanent pore-water pressures. Therefore, the test results furnish only a rough indication of the influence of the rheological component of strength. The time-strength ratios cited above are 0.86 to 0.88 for τ_f and an average of 0.75 for c_e . These ratios indicate that the variation of the rheological component with time reduces τ_f by 12 to 14 per cent and c_e by 25 per cent when the test duration is increased from 3 minutes to 600 minutes or 200-fold. The rheological decrease in strength is not complete at a test duration of 600 minutes, but its ultimate value cannot be determined by means of available test data or without further assumptions.

Investigations discussed in the following subsection show that the compressive strength of saturated clays decreases linearly with the logarithm of time, Fig. 33 and Eq. 48. It is probable that this relation also applies to the decrease with time of τ_f and c_e . Values of the coefficient ρ_3 in Eq. 48 are obtained by dividing the reduction percentages, given in the preceding paragraph, by (log 600 - log 3) = log 200 = 2.3, which yields 0.05 to 0.06 for τ_f and 0.11 for c_e . Values of c_e at the time t , designated by c_{et} , can then be determined by the equation

$$c_{et} = c_{e3}(1 - 0.11 \log \frac{t}{3}) \tag{47}$$

where t is the time in minutes, and c_{e3} is the value of c_e for $t = 3$ minutes. Numerical values of c_{et}/c_{e3} obtained by this equation are shown in Table 3. It is not yet known whether Eqs. 47 and 48 are valid for very high values of t . It is probable that c_e with time approaches a limiting value, c_u , which is the actual cohesion component. The rheological component, c_v , at the time t is then determined by $c_v = c_{et} - c_u$. For the purpose of illustration it may be assumed that c_u is equal to the 10-year value of c_{et} . Corresponding values of c_u/c_{et} and c_v/c_{et} are also shown in Table 3, and it is seen that c_u is 43 per cent and c_v is 57 per cent of c_e in a common slow test with t = 10 hours.

Table 3

Influence of the Rheological Component for Vienna Clay

	Time t				
	3 min	10 hr	10 Days	1 Year	10 Years
c_{et}/c_{e3}	1.00	0.75	0.60	0.43	0.32
c_u/c_{et}	0.32	0.43	0.54	0.75	1.00
c_v/c_{et}	0.68	0.57	0.46	0.25	0.00

The data in the preceding paragraph and Table 3 are speculative and are presented in order to demonstrate the influence of the rheological component and a method when adequate test data are available. In the example cited, c_u forms a substantial part of c_e determined by slow tests, but c_u may be close to zero for other clays with the same time-dependent decrease in strength but having smaller values of the parameters for the effective cohesion component.

The Long-Term Compressive Strength

CASAGRANDE and WILSON (1949, 1950) have made extensive and very thorough investigations of the long-term compressive strength of clays and clay-shales at constant water content and under sustained loads. It was found that the strength decreases linearly with the logarithm of time when the clays are saturated. This approximate relation, which has been verified by other investigators, is illustrated in Fig. 33 and can be expressed analytically by

$$q_t = q_a - \ddot{q} \log(t/t_a) = q_a[1 - \rho_a \log(t/t_a)] \tag{48}$$

or as suggested by GOLDSTEIN (1957)

$$q_t = \ddot{q} \log(t_o/t) \tag{49}$$

where t_o is the time corresponding to zero strength, and $\ddot{q}$ is the decrease in strength for a logarithmic cycle of time or the strength for $t = 0.1\ t_o$. Eq. 49 is advantageous for further mathematical developments, whereas Eq. 48 offers an opportunity to characterize the decrease in strength by a single parameter $\rho_a = \ddot{q}/q_a$, which may be called the coefficient of rheological decrease in strength. This coefficient varies with the reference strength q_a and time t_a , and its value, ρ_b , corresponding to another reference strength q_b and time t_b is easily determined by the relation $\ddot{q} = \rho_a q_a = \rho_b q_b$, or

$$\rho_b = \rho_a(q_a/q_b) = \rho_a/[1 - \rho_a \log(t_b/t_a)] \tag{50}$$

Casagrande and Wilson discuss both transient and long-term strengths, and use the one-minute strength as a reference value for determination of the coefficient of decrease in strength, which in the following will be designated by

ρ_1 . Several other investigators are primarily concerned with the long-term strength, in which case it is more significant to use a reference strength corresponding to that obtained in normal slow tests. The writer suggests that the 1000-minute strength be used as the reference strength, and that the corresponding value of the coefficient of decrease in strength be designated by ρ_m . According to Eq. 50, the relation between ρ_1 and ρ_m is

$$\rho_m = \rho_1/(1 - 3\rho_1) \tag{51}$$

The data presented by CASAGRANDE and WILSON (1951) yield the following coefficients of decrease of the undrained strength of clays and clay-shales with time

General range $\rho_1 = 0.04$ to 0.09 $\rho_m = 0.05$ to 0.13

Cucaracha clay-shale $\rho_1 = 0.07$ to 0.19 $\rho_m = 0.09$ to 0.46

BJERRUM-SIMONS-TORBLAA (1958) investigated strength-time relations for a normally consolidated marine clay in its undisturbed state, using controlled-strain type of loading. The results of consolidated undrained tests agree with Eq. 48, provided the test duration is greater than one hour, and yield $\rho_m = 0.06$ to 0.07. The authors also found that the pore-water pressure at failure increases with the test duration for normally consolidated test specimens, and that this increase in pore-water pressure and corresponding decrease in effective stresses can explain a part but not all of the decrease in strength with increasing test duration. The test data were evaluated by the method represented by Eq. 9-B, and the results are summarized in Fig. 34. It is seen that the angle of shear strength, ϕ' , decreases with increasing test duration. The lines OA and OB correspond to the shear strength lines and represent changes in both effective stresses and void ratios. As mentioned by the authors, the data obtained were not sufficient for computation of the effective friction and cohesion parameters and determination whether the decrease in strength with time affects not only the cohesion - but also the friction parameter.

BJERRUM-SIMONS-TORBLAA (1958) also conducted consolidated drained triaxial tests on the marine clay, and found that the strength is independent of the test duration when the latter is greater than one day. The authors suggest that the decrease of the rheological component in this case is compensated by an increase of the cohesion component, caused by secondary consolidation and decrease in void ratio.

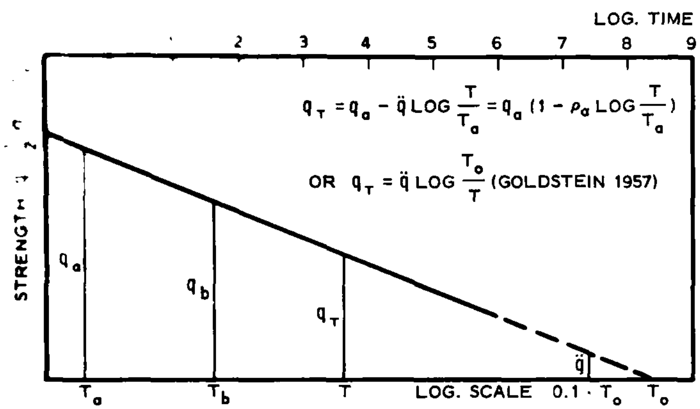


FIG. 33. UNDRAINED STRENGTH VERSUS TEST DURATION

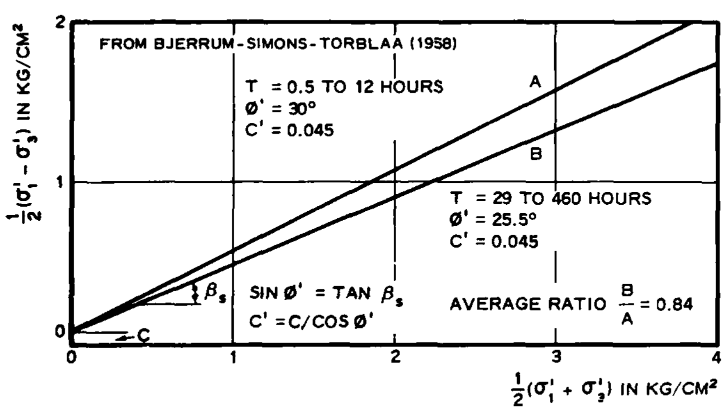


FIG. 34. STRENGTH VERSUS EFFECTIVE STRESS AND TIME

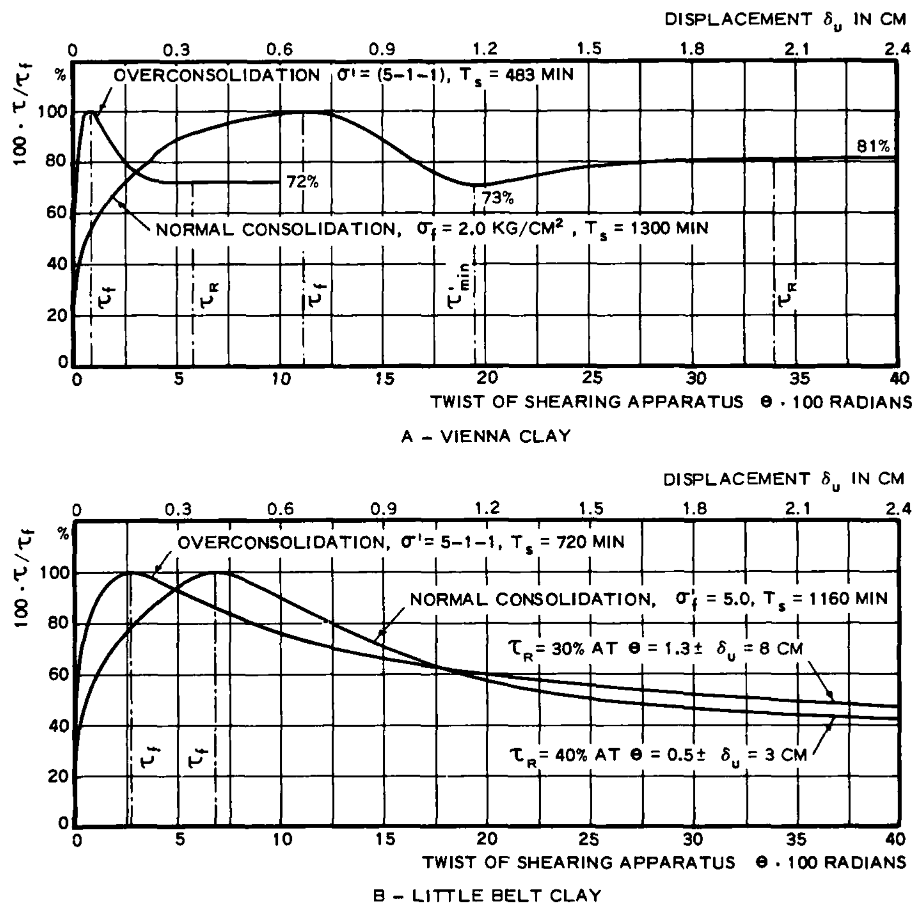


FIG. 35. STRESS-STRAIN DIAGRAMS FOR SHEAR BEFORE AND AFTER FAILURE

BISHOP and HENKEL (1957) investigated the long-term strength of Weald clay and London clay, and the data obtained yield the following values of ρ_m

London clay, normal consolidation, drained test, $\rho_m = 0.06$

Weald clay, normal consolidation, drained test, $\rho_m = 0.04$

Weald clay, overconsolidation, drained test, $\rho_m = 0.04$

Weald clay, normal consolidation, undrained test, $\rho_m = 0.06$

It is surprising that the same values of ρ_m were obtained for normally consolidated and overconsolidated Weald clay, which may indicate that the influence of secondary volume changes was negligible or that compensating factors were active. The higher value of ρ_m obtained in the undrained tests reflects the influence of increasing pore-water pressures.

GOLDSTEIN (1957) presents time-strength data for an unspecified clay which yield $\rho_m = 0.09$ to 0.13. Goldstein defines the failure strain and time, caused by a given stress, as that at which the rate of strain starts to increase, and he found that this strain is independent of the test duration for a given soil, state of consolidation, and testing procedure. Therefore, the time of failure for a given stress can be determined by extrapolating the time-strain curve for this stress to the failure strain. In a subsequent discussion GOLDSTEIN (1958) also proposed amended analytical expressions for the stress-strain-time relations and derived corresponding expressions for the long-term strength.

Some of the data presented by CASAGRANDE and WILSON (1949, 1951) show but little influence of the test duration on the failure strain, whereas other data indicate an increase in failure strain with increasing test duration. BJERRUM-SIMONS-TORBLAA (1958) found a definite decrease in failure strain with increasing test duration. WHITMAN (1960) has conducted extensive investigations of the influence of high rates of loading on strength and strain and found that the failure strain of plastic soils is nearly independent of the test duration, whereas that of brittle soils decreases with increasing test duration.

Some of the previously mentioned tests for determination of the time-dependent decrease in strength and the failure strain were performed with controlled stress loading and others with a controlled rate of strain. It is possible that results obtained with the two types of loading may not be directly comparable because of differences in the rate of strain at failure for a given test duration. There may also be differences in the definition of the

failure strain, and attention is called to the previously mentioned definition by GOLDSTEIN (1957).

Tests on clays at constant water content yield one rate for the decrease in strength with time, and another rate is obtained by drained tests. It is possible that the decrease in strength of natural clay deposits may have a magnitude which is between the two rates, because a slight migration of pore water may cause a considerable change of the pore-water pressure in the failure zone. A migration of pore water takes place in undrained compression tests but it may not be of the same magnitude as that which occurs in natural deposits over long periods of time.

The data discussed in this section refer to saturated clays. As shown by CASAGRANDE and WILSON (1951), the strength of partially saturated, compacted cohesive soils, held at constant water content, may initially decrease slightly and thereafter increase with time.

Summary and Comments

The principal causes of continuing shear deformations and time-dependent changes in strength of cohesive soils may be summarized as follows:

- a. Primary changes in pore-water pressure after an increase in shear stress. An increase in shear stress causes a decrease in pore-water pressure in strongly overconsolidated clays, but this decrease may be equalized in the course of time by migration of pore water, which in turn produces a gradual increase in void ratio and decrease in effective stresses, and a corresponding decrease in strength and increase in strain.
- b. Secondary changes in pore-water pressure caused by a sustained increase in shear stress. It has been demonstrated that the pore-water pressure of a normally consolidated clay increases with time when triaxial test specimens are held at constant water content and subjected to a sustained axial load. The behavior of strongly overconsolidated clays under similar conditions is not yet known.
- c. The above-mentioned effect of an increase in shear stresses may initially be increased or decreased by a concurrent change in the effective, mean normal stress $(\sigma_1' + \sigma_2' + \sigma_3')/3$.
- d. The existence of a viscous or rheological component of the shear strength and its variation with time and the rate of deformation.

_e_. Thixotropic changes in strength with the rate of deformation, which often may be difficult to separate from concurrent changes in the rheological component of strength.

_f_. Structural disturbance of the clay in the zone of current or former failures and corresponding transient and permanent changes in shear strength.

The design of many foundation structures is governed by the deformations rather than the limiting strength of the foundation soils, but adequate experimental rheological data are generally lacking, and the restrictions imposed by the deformations are often taken into consideration by using allowable strengths which correspond to small but arbitrarily selected strains on the stress-strain diagrams obtained in standard laboratory strength tests. This procedure is a practical expedient which should be amplified and delimited by detailed and systematic research, utilizing the methods and theories of plasticity and rheology. However, modifications may be required, since these theories have been developed for materials with volume change characteristics which are different from those of soils, especially in regard to the volume changes caused by pure shear stresses. Investigations of the long-term rheological properties of clays are by nature time-consuming, but basic research into the problem, and correlations with index properties and the state of consolidation of clays, will undoubtedly lead to short-term tests or methods for making reliable estimates.

8. CHANGES IN SHEAR STRENGTH AFTER FAILURE

Knowledge of the changes in shear strength and of the shape of the stress-strain curves for soils after failure is of practical importance for estimating the effects of progressive failure or previous disturbance of the soil in stability and bearing capacity problems and for various special purposes. Determination of the stress-strain curve after failure may be difficult when large deformations are involved, which generally is the case. The accuracy of standard box shear tests, unconfined and triaxial compression tests decreases with increasing deformations because of changes in the effective cross section of the test specimen. The ROSCOE (1953) direct shear apparatus maintains a constant cross section of the test specimen and can be used for relatively large but nevertheless limited deformations. Deformations of any magnitude can be obtained with the torsion ring shear apparatus, and the cross section of the test specimen remains constant, but the deformations after failure are confined to a relatively thin zone. Limited data on the stress-strain curve after failure can also be obtained by means of vane shear tests.

The following data on changes in shear strength after failure of Vienna clay and Little Belt clay were obtained by means of the torsion ring shear apparatus shown schematically in Fig. 11. The tests were performed by incremental changes in torque or shear stress. Points on the stress-strain curves after failure were obtained by rapidly reducing the shear load until the deformations ceased and immediately thereafter increasing the shear load until failure again occurred. The tests were in fact conducted as slow drained tests before failure, and as fairly rapid tests with short interruptions after failure. However, rest periods between subsequent repetitive loadings permitted equalization of static pore-water pressures. The increase in deformations with increasing distance from the center of the test specimens was taken into consideration by the method of evaluation of the test results described in previous publications, HVORSLEV (1937, 1939).

Tests on Vienna Clay

The residual strength. Typical stress-twist or stress-displacement curves are shown in Fig. 35-A. The curve for normally consolidated Vienna clay has a transient minimum of 73 per cent of the maximum strength, which undoubtedly is caused by development of excess pore-water pressures after

failure, and the subsequent increase in strength to 81 per cent is due to partial equalization of these pore-water pressures. An increase in shear stresses tends to cause a decrease in void ratio of normally consolidated clays, and an additional decrease in void ratio or increase in pore-water pressures occurs after failure. In contrast thereto, strongly overconsolidated clays undergo an increase in void ratio or a decrease in pore-water pressure before and after failure. Therefore, the stress-twist curve for strongly overconsolidated Vienna clay does not exhibit a transient minimum value of the shear strength during continuous and fairly uniform deformations after failure, and the relative residual shear strength, 72 per cent, is smaller than that for the normally consolidated clay.

Regain of shear strength. Upon reaching an apparent residual value of the shear strength, the above-mentioned shear tests were interrupted by reducing the shear load, and the tests were repeated after rest periods of increasing length. The results obtained for the normally consolidated Vienna clay are shown in Fig. 36. It may be noted that the original shear strength of this clay was 1.00 kg/cm^2, shown as 100 per cent in Fig. 35-A. The residual strength increased from the transient minimum of 0.73 kg/cm^2 to 0.83 kg/cm^2 in less than 82 minutes and remained constant thereafter. After a rest period of 10 minutes the shear strength had increased from the residual strength to 99 per cent of the original strength; thereafter the strength increased linearly with the logarithm of time and reached 1.11 kg/cm^2 after a rest period of 238 hours. It is estimated that the decrease in water content after failure was about 1.0 per cent, which corresponds to an increase of 0.09 kg/cm^2 in the effective cohesion component.

A similar increase in strength upon cessation of deformations was also observed for strongly overconsolidated Vienna clay, but the rest periods were not of sufficient length to determine definitely whether the original strength ultimately would be regained. The overconsolidated clay underwent a small increase in void ratio after failure, and it is probable that the original shear strength would not be fully regained by a longer period of rest.

The results of these tests indicate that the decrease in shear strength after failure primarily is a thixotropic phenomenon for remolded Vienna clay, and that the structure of the remolded clay has not been altered appreciably by the failure.

Residual strength parameters. The final residual strengths obtained for normally consolidated and overconsolidated test specimens of Vienna clay form

a shear strength diagram similar to that shown in Fig. 21. The friction and cohesion parameters were determined as shown in Fig. 37, where σ'_e is the equivalent consolidation pressure corresponding to the final water content in the failure zone, and σ'_f is assumed to be equal to the total normal stress, σ'_f . A comparison of Figs. 24-A and 37 shows that failure does not affect the effective angle of internal friction but reduces the effective cohesion parameter. It was originally believed that the residual strength parameters thus determined were reliable, because the void ratios apparently had reached stable values and the soil structure had not been changed appreciably. MITCHELL (1960) and others have recently shown that changes in the pore-water pressure take place during thixotropic changes in strength, in which case the values of σ_f used in Fig. 37 are not the true effective stresses. However, Mitchell also demonstrated that thixotropic changes do not affect the strength expressed in terms of effective stresses. Therefore, the strength parameters shown in Fig. 24-A also apply to the residual strength, and a comparison with Fig. 37 indicates that the transient pore-water pressure caused by a thixotropic decrease in strength is a constant for a given void ratio, but this applies only to the final residual strength and not to the transient minimum strength exhibited by remolded and normally consolidated Vienna clay. As seen in Fig. 37, available data for determination of the residual strength parameters are very meager, and additional investigations are needed.

Tests on Little Belt Clay

The stress-twist diagrams in Fig. 35-B show a much greater decrease in strength after failure than that obtained for Vienna clay, and it was found that the regain in strength upon cessation of the deformations was incomplete. The strength of the overconsolidated test specimen increased from the residual value of 30 per cent to 37 per cent during a rest period of 4 days and to 55 per cent in 60 days. The determination of the residual strength parameters in Fig. 37 should be corrected for pore-water pressures corresponding to the thixotropic change in strength, which in this case accounts for only a part of the total decrease in strength. It is evident that failure causes a decrease of both the friction and cohesion parameters and also a permanent change in the soil structure. The failure surfaces were distinct and smooth but not glossy.

As previously mentioned, the curvature of the shear strength line for normally consolidated Little Belt clay, Fig. 22, was not caused by too short

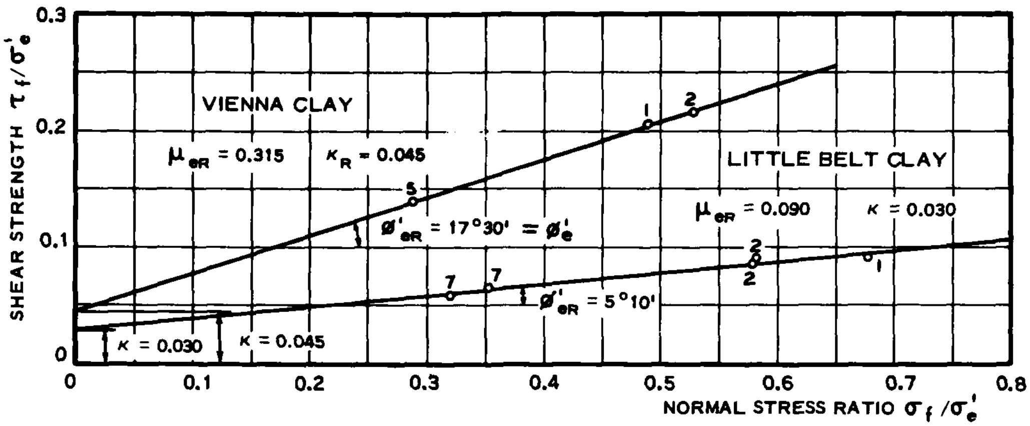
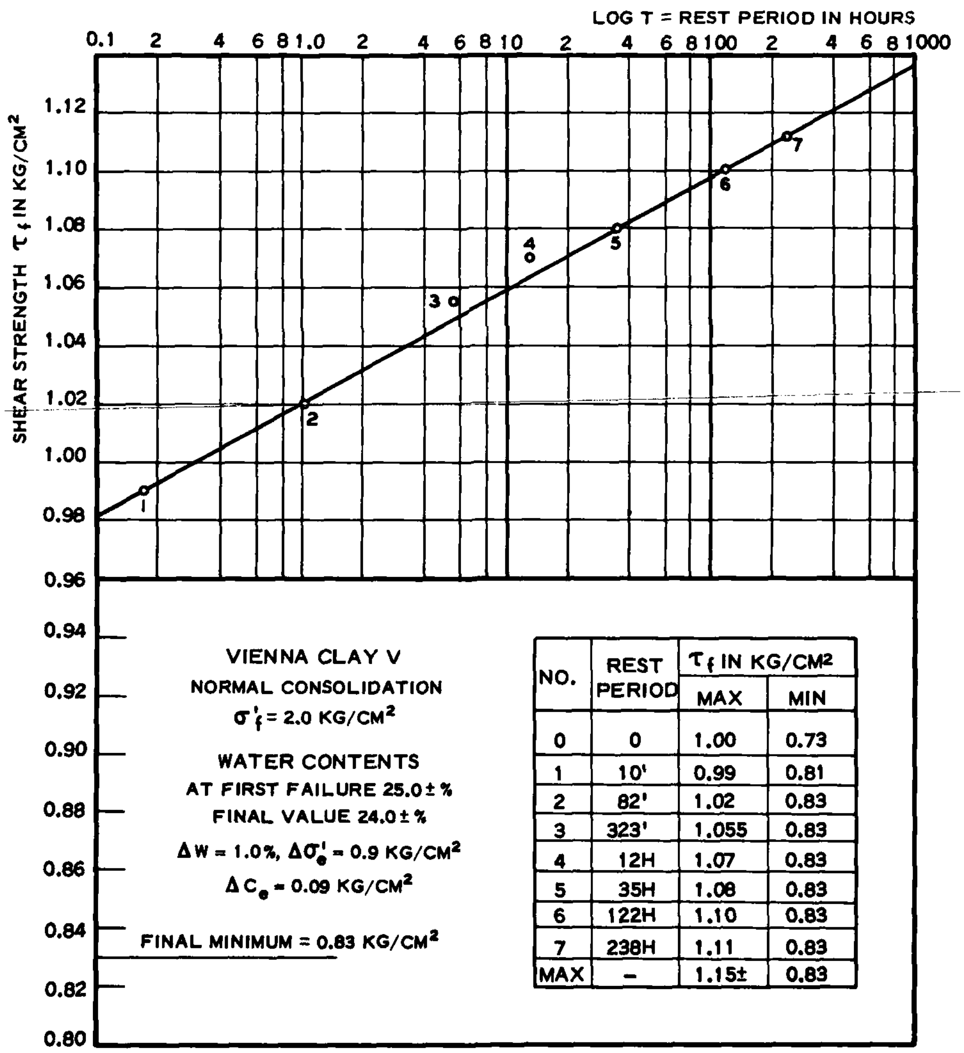


FIG. 37. FRICTION AND COHESION PARAMETERS FOR RESIDUAL STRENGTH



NO.	REST PERIOD	τ_f IN KG/CM² MAX	MIN
0	0	1.00	0.73
1	10'	0.99	0.81
2	82'	1.02	0.83
3	323'	1.055	0.83
4	12H	1.07	0.83
5	35H	1.08	0.83
6	122H	1.10	0.83
7	238H	1.11	0.83
MAX	–	1.15±	0.83

FIG. 36. REGAIN OF SHEAR STRENGTH WITH TIME FOR VIENNA CLAY

a duration of the tests. It is possible that a part of the above-mentioned structural disturbance of Little Belt clay occurs before failure, and that this disturbance increases with increasing normal pressure and may account for the curvature of the shear strength line. The fact that structural disturbance occurs under the above-mentioned circumstances may also indicate that it is not possible to obtain complete dispersion of a highly plastic clay by mechanical remolding.

Other Investigations

TIEDEMANN (1937) developed a torsion ring shear apparatus similar in principle to that shown in Fig. 11 but with a different design of the details. Tests were performed on four remolded and undisturbed clays, ranging from silty clays to highly plastic clays. The test specimens were normally consolidated, and the testing procedure was similar to that described for Vienna and Little Belt clays. The residual strengths varied from 20 per cent to 60 per cent of the maximum strength and were attained after displacements of 10 to 14 cm.

As previously mentioned, a torsion ring shear apparatus with both incremental stress and controlled strain types of loading has been developed by the USAE Waterways Experiment Station. A series of tests on Atlantic Muck, a highly organic clay, was performed with this apparatus. The test results are presented in a detailed report by the WATERWAYS EXPERIMENT STATION (1951) and are summarized by HVORSLEV and KAUFMAN (1951). The test specimens were undisturbed and normally consolidated or overconsolidated under various pressures. Controlled strain type of loading was used, and the starting rate produced failure in 3 to 6 minutes; thereafter the rate of displacement was increased five times, but the total duration of the tests was nevertheless sufficient to permit appreciable changes in pore-water pressures and void ratio, which made the evaluation of the test results rather difficult. Transient minimum strengths after failure were observed for normally consolidated and lightly overconsolidated test specimens. The residual shear strengths ranged from 30 per cent to 90 per cent of the maximum strengths and were attained after displacements of 3 to 10 cm, according to the state of consolidation of the clay. Slickensided failure surfaces were produced after large displacements, Fig. 3-B, but only when the normal stress was equal to or greater than 1.0 kg/cm^2.

The form of stress-displacement curves for soils, before and after

failure, is important for estimation of the tractive power of vehicles. BEKKER (1956, pp. 263-273) has proposed mathematical expressions for complete stress-displacement curves, originally obtained by tests resembling direct box shear tests. Improved equipment for in-situ determination of the stress-displacement curves has recently been developed by the Land Locomotion Research Branch, Ordnance Tank-Automotive Command, Department of the Army; see PAVLICS (1958) and VINCENT (1959). The apparatus, called a bevameter, consists of an annular plate with ribs or grousers which is pressed into the soil, subjected to a desired normal load, and rotated.

Indications of the form of stress-displacement curves before and after failure can also be obtained by vane tests. These tests are primarily used for determination of maximum and residual shear strengths of clays in situ, and relatively few stress-displacement diagrams have been published. MARSAL (1957) presents stress-displacement diagrams obtained by vane tests in the volcanic clays of Mexico City. The residual strengths ranged from practically zero to 30 per cent of the maximum strengths and were attained after displacements of 8 to 12 cm. Neither the effective nor the total normal stresses existing during vane tests are definitely known.

The large deformations required to attain the residual shear strength of plastic clays cannot be produced in triaxial or unconfined compression tests. However, these tests may be used for determination of complete stress-strain curves of brittle clays and clay shales. Examples of such curves are shown in a paper by CASAGRANDE and SHANNON (1949).

Influence of the Rate of Deformation

Published data on the influence of the rate of deformation on the residual shear strength and the shape of the stress-strain curve after failure are very limited. A single test for this purpose has been made at the WATERWAYS EXPERIMENT STATION (1951) and fragmentary data may be obtained from some triaxial tests and vane shear tests. An increase in the rate of deformation increases the rheological or viscous component of strength, but it also increases the thixotropic decrease in strength. Proceeding from very low to very high rates of deformation, the combined effect is at first a decrease in residual strength; thereafter the residual strength appears to remain fairly constant over a rather wide range of rates of deformation; finally the residual strength increases at very high rates of deformation. Migration of water from or to the shear zone increases the residual strength of normally

consolidated clays and decreases that of strongly overconsolidated clays; therefore, the slope of the stress-deformation curve after failure is also affected by time and the rate of deformation.

When controlled or incremental stress type of loading is used, the rate of deformation increases with increasing shear stress, Fig. 30; under the last load increment, this rate may remain fairly constant for a considerable period, but a progressive increase in the rate of deformation occurs at and after failure with corresponding thixotropic changes in strength and changes in the pore-water pressure. This type of loading tends to produce a decrease in shear strength after failure of normally consolidated and lightly overconsolidated clays, but such a decrease in strength may not always occur in undrained tests on strongly overconsolidated clays.

Loading by a very low and constant rate of strain before and after failure may reduce the thixotropic decrease in strength after failure to an insignificant amount, and it facilitates equalization of pore-water pressures and changes in void ratio. As a consequence, this type and rate of loading may not produce a significant decrease in strength after failure in undrained tests on overconsolidated clays or in drained tests on normally consolidated and lightly overconsolidated clays, unless failure causes a permanent or actual disturbance of the soil structure. However, this type and rate of loading tends to produce a decrease in strength after failure in drained tests on strongly overconsolidated clays, WROTH (1958).

There is need of detailed investigations of the influence of the rate of deformation on the shear strength after failure, and there is an especially great need of comparisons of the results obtained in very slow tests with controlled stress and controlled strain types of loading.

9. TRIAXIAL CONSOLIDATION AND FAILURE CONDITIONS

The data and concepts presented in the foregoing sections are primarily based on the results of confined consolidation tests and direct box or torsion shear tests. Some comments on consolidation and failure relations obtained by triaxial tests are made in this section, but these comments should not be interpreted as a complete review or summary of the subject.

Triaxial Stress Notation

In evaluation of the results of triaxial tests, it is generally assumed that the stress distribution is uniform. Referring to Fig. 38-A, $\sigma_z' = \sigma_a'$ is the effective axial stress and $\sigma_x' = \sigma_y' = \sigma_r'$ are the effective lateral stresses, and these are assumed to be the principal stresses. In recent years the test results are often expressed in terms of the first, second, and third stress invariants

$$J_1 = \sigma_1 + \sigma_2 + \sigma_3 \tag{52-A}$$

$$J_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 \tag{52-B}$$

$$J_3 = \sigma_1 \sigma_2 \sigma_3 \tag{52-C}$$

or the equivalent octahedral stresses

$$\sigma_{oct} = \frac{1}{3} J_1 = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3) = p \tag{53-A}$$

$$\tau_{oct} = \frac{1}{3} \sqrt{2(J_1^2 - 3J_2)} = \frac{1}{3} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \tag{53-B}$$

The octahedral normal stress is also called the mean normal stress and designated by p . For effective stresses in triaxial tests these formulas take the form

$$\sigma_{oct}' = \frac{1}{3} (\sigma_a' + 2\sigma_r') = p' \tag{54-A}$$

and

$$\tau_{oct} = \frac{\sqrt{2}}{3} (\sigma_1' - \sigma_3') \tag{54-B}$$

where $\sigma_1' = \sigma_a'$ for compression tests and $\sigma_1' = \sigma_r'$ for extension tests.

Sources of Error in Triaxial Tests

Changes in external forces caused by piston friction and by the rubber

membrane have been investigated in considerable detail and are now fairly well known. Some difficulties are still encountered in accurate measurements of changes in diameter and volume of the test specimen during a test.

The most disturbing source of error is the influence of the end restraint or friction between the ends of the loading plates and the test specimen, which may cause a nonuniform distribution of stresses, strains, volume changes, and pore-water pressures in the triaxial test specimen. A brief review of theoretical and experimental investigations of the influence of end restraint and other sources of error is contained in a discussion by the writer, HVORSLEV (1957), and details are presented in papers of this conference by WHITMAN (1960), BISHOP-ALPAN-BLIGHT-DONALD (1960), and SHOCKLEY-AHLVIN (1960). Nonuniformity exists not only in axial but also in radial directions, and the pattern of the nonuniformities varies with the external stress conditions and the state of consolidation of the test specimen; therefore, this pattern changes during a test. Direct determination of pore-water pressures, water contents, or volume changes in various parts of a test specimen show that internal migration of pore water takes place during undrained tests and also that a volume decrease in one part of a drained test specimen may occur simultaneously with a volume decrease in another part of the specimen. The experimentally determined degree of nonuniformity at large strains is much larger than that obtained by a theoretical analysis of the influence of end restraint, but the causes of this additional nonuniformity have not yet been definitely determined.

The influence of the above-mentioned sources of error may not be too serious when the strains are small, but the difficulties encountered in proper evaluation of the test results increase rapidly with increasing strains. Further investigations of the magnitude of the nonuniformities under various stress, strain, and consolidation conditions are needed in order to develop appropriate methods of correction or to establish practical limits for the reliability of results of triaxial consolidation and strength tests.

Definitions of Triaxial Failure and Strength

The shear strength determined by direct shear tests is defined as the peak strength, τ_f , in the stress-strain diagram, Fig. 4. A similar definition may also be used in triaxial tests by considering the stress conditions on certain assumed failure or critical planes. Investigation of the stress conditions on such planes is very useful, as in form of the vector curves by

CASAGRANDE and WILSON (1953), but it does involve assumptions in regard to the inclination of the planes. Such assumptions are avoided by defining the state of failure in triaxial tests as the peak value of $(\sigma_1 - \sigma_3) = (\sigma_1' - \sigma_3')$, and this definition is used in this paper unless otherwise noted.

The peak values of τ and $(\sigma_1' - \sigma_3')$ occur for the same stress condition and strain when the assumed inclination of the critical plane is the same throughout a test. However, these peak values may occur at different strains when the inclination of the critical plane is assumed to be that furnishing the maximum obliquity of the resultant stress and varies with the principal stress ratio; TAYLOR (1948, 1955).

The peak value of the effective principal stress ratio, σ_1'/σ_3', has been and is still being used to define the state of failure in triaxial tests, although this definition may lead to inconsistencies when the soil also is assumed to conform to the Mohr-Coulomb failure criterion and when the Mohr envelope does not pass through the origin; TAYLOR (1950). Eq. 9-A may be written in the following form

$$\frac{\sigma_1'}{\sigma_3'} = \frac{(\sigma_1' - \sigma_3') - 2c' \cos \phi'}{\sigma_3' \sin \phi'} \qquad (9\text{-F})$$

which shows that the maximum values of (σ_1'/σ_3') and $(\sigma_1' - \sigma_3')$ occur simultaneously or at the same strain when σ_3' is held constant during the test; see HILF and GIBBS (1957). It is possible to maintain a constant value of σ_3' in drained tests and also in some undrained tests on partially saturated soils, but σ_3' cannot be held constant in undrained tests on fully saturated clays. During undrained tests on saturated and normally consolidated clays, σ_3' decreases with increasing strain and the maximum value of (σ_1'/σ_3') may then occur after the maximum value of $(\sigma_1' - \sigma_3')$ is attained. On the other hand, σ_3' increases during undrained tests on saturated and strongly overconsolidated clays, and a peak value of (σ_1'/σ_3') may then occur before the peak value of $(\sigma_1' - \sigma_3')$ is reached. In general, definition of the state of failure by the peak value (σ_1'/σ_3') tends to produce greater values of ϕ', and smaller values of c' than those obtained when failure is defined by the maximum value of $(\sigma_1' - \sigma_3')$.

It is often observed that $(\sigma_1' - \sigma_3')$ continues to increase whereas (σ_1'/σ_3') remains fairly constant during the last part of undrained triaxial tests on strongly overconsolidated clays. A maximum value of $(\sigma_1' - \sigma_3')$ may not be reached at strains ordinarily used or attainable in triaxial tests, but

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PHYSICAL COMPONENTS OF THE SHEAR STRENGTH OF SATURATED CLAYS.(U)
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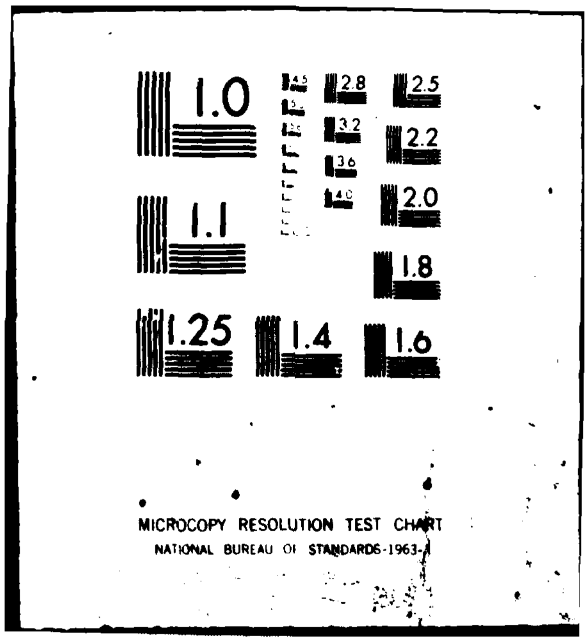
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MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

a maximum value does exist according to the critical void ratio concept of ROSCOE-SCHOFIELD-WROTH (1958), which is discussed later in this section. The changing stress conditions during the last part of such a test are very close to those of the failure envelope, and the peak value of (σ_1'/σ_3') or any value of $(\sigma_1' - \sigma_3')$ during the last part of the test, combined with the corresponding value of σ_3', may be used for expressing the failure conditions in terms of effective stresses.

Two definitions of comparative strengths are used in evaluating the influence of various triaxial stress conditions on the strength of clays. The first may be termed the <u>constant volume strength</u> and is the peak value of $(\sigma_1 - \sigma_3)$ for the same void ratio but varying stress conditions. The second type of comparative strengths may be called the <u>effective stress strength</u> and is the peak value of $(\sigma_1' - \sigma_3')$ for equal values of σ_3', or, considering stresses on assumed failure planes, value of τ_f for equal values of σ_f'. Comparative effective stress strengths indicate the relative position of the failure envelopes or shear strength lines for various stress conditions.

These comments on the state of failure are generalizations and simplifications of actual conditions, and there is need of further and more detailed investigations of the definitions of the state of failure. These investigations should take into consideration the influence of the type and rate of loading, the development of surface cracks, increasing nonuniformities in the test specimen with increasing strains, the yield and flow limits proposed by GEUZE (1960) and the possible existence of a limiting failure strain as suggested by GOLDSTEIN (1957).

The Rendulic Diagram

Basic experimental relations between triaxial stress conditions, void ratios, and pore-water pressures for normally consolidated clays were first established by RENDULIC (1936, 1937). The importance of this classic work has remained practically disregarded until recently verified and amplified through a series of very valuable investigations at the University of London, HENKEL (1958, 1959, 1960). The tests by Rendulic were performed in Vienna clay, and the most complete series of tests reported by Henkel were made on Weald clay. The two clays are quite similar, and their index properties and the molding water contents used are given in the following table.

Table 4

Index and Molding Water Contents

	Vienna Clay	Weald Clay
Liquid limit	47.6	43
Plastic limit	22.8	18
Molding water content	27.5	34
Liquidity index	0.19	0.64

Rendulic tests. Both drained and undrained, compression and extension tests were performed on Vienna clay. The test specimens were drained by a central core of a sand and mica mixture, and the pore-water pressures measured were those existing in this core. RENDULIC (1937) states that corrections were not made for the influence of changes in cross section of the test specimen with increasing strains. Therefore, data obtained for large strains, including failure conditions, are not reliable. The low initial or molding water content should be noted, since it undoubtedly affected the test results. RENDULIC (1936) states that the molding water content corresponds to a pressure of 6 kg/cm^2 in confined consolidation, starting at the liquid limit. This statement is probably a misprint, and it is not repeated in the 1937 and more complete version of the paper. The initial water content is equivalent to a void ratio of 0.76, and it is seen in Fig. 10 that it corresponds to an equivalent consolidation pressure of about 2 kg/cm^2.

Henkel tests. A very comprehensive and carefully executed series of triaxial tests on Weald clay were performed at the University of London by Henkel and graduate students over a period of five years. This series included drained and undrained tests, compression and extension tests, various types of drained tests, and tests on both normally consolidated and overconsolidated test specimens. A similar series of tests were performed on London clay but were confined to compression-type tests. It should be noted that the molding water content and the corresponding value of the liquidity index were much higher than those used by Rendulic. The test specimens were drained at the end, and the pore-water pressures were measured there, but drainage was aided by strips of filter paper on the surface of the test specimen. There were minor differences in the consolidation characteristics of the batches of clay used at the beginning and end of the test series.

The Rendulic diagram. The pressure-void ratio relations determined by

drained tests may be plotted in the conventional semilogarithmic diagram, and an example of data obtained by Rendulic for all-round or hydrostatic pressure is shown in Fig. 10. The points lie on a straight line but, because of the low molding water content, this line is considerably below and has a flatter slope than the virgin consolidation diagram obtained by confined consolidation and an initial water content close to the liquid limit.

Rendulic devised an ingenious method for comprehensive graphical representation of relations between void ratios and stress conditions in triaxial tests. Points representing these stress conditions lie in the plane of symmetry, BOC in Fig. 38-A, where the ordinates are σ_a' and the abscissae are $\sigma_r'\sqrt{2}$. In this plane Rendulic plotted contours of equal water content or void ratio, as shown in Fig. 38-B for states of normal consolidation of Weald clay. The line DD and lines parallel thereto represent states of constant values of the mean effective stress, p' , or the invariant J_1' , and these lines are tangent to the constant void ratio contours at their intersection with the line OA, which represents states of all-round stress or $\sigma_a' = \sigma_r'$. Rendulic found fairly good agreement between the results of drained and undrained tests, and he concluded that any point in the diagram represents a unique relation between void ratio and stress condition, which is independent of the stress path provided this path does not cause a temporary decrease in void ratio.

The above-mentioned results and conclusions by Rendulic have in general been verified by Henkel. The shapes of the contours for drained and undrained tests are nearly identical, but there are in some cases minor differences in the stresses or the position of the contours. As yet it is uncertain whether these differences are systematic or merely represent unavoidable differences in the properties of the test specimens, but the possibility exists that further investigations may show that the stress paths in some cases may affect the position of the contours.

Values of (σ_a' , σ_r') at failure of Weald clay, defined as the maximum values of $(\sigma_1' - \sigma_3')$, are also shown in Fig. 38-B and form two straight lines, OE and OF, which are envelopes to the constant void ratio contours and pass through the point of origin when the test specimens are normally consolidated. These lines correspond to the Mohr-Coulomb failure criterion and an angle of inclination $\phi_m' = 22^\circ 10'$, and they were found by Henkel to be independent of the type of test, drained or undrained, and various combinations of increases and decreases of σ_a' and σ_r' .

The unified Rendulic diagram. The constant water content contours for a saturated and normally consolidated clay appear to have nearly identical shapes for water contents appreciably smaller than the molding water content. Therefore, a family of such contours can be replaced with a single curve which is a function of the ratios σ'_a/σ'_c and σ'_r/σ'_c, where σ'_c is the stress at the intersection of the contour and the line OA; that is, σ'_c is the equivalent consolidation pressure for all-round or hydrostatic pressure. Such unified Rendulic diagrams for normally consolidated Vienna clay and Weald clay are shown in Fig. 38-C and are for convenience of operation plotted in the XZ plane instead of the plane of symmetry. In the XZ plane the line OA has a slope of 1:1 and the line DD a slope of 1:2. The contours appear in a slightly distorted form, but they are still tangent to the line DD in point A. There are some irregular variations in the shape of individual constant water content contours obtained by Rendulic, and the unified contour shown in Fig. 38-C represents the average shape of several contours obtained for all-round consolidation pressures close to 4.0 kg/cm^2. The unified diagram cannot fully replace the family of contours, but it is convenient for a general discussion of relations and for comparison of characteristics of different clays.

Failure envelopes for Weald clay are also shown in Fig. 38-C but as functions of σ'_a and σ'_r instead of the above-mentioned ratios. The corresponding dashed lines for Vienna clay are envelopes for the contours obtained by Rendulic, but they are not the actual failure envelopes since the test data were not corrected for the influence of changes in cross section of the test specimen during the tests. Corrections for changes in cross section of the test specimen, estimated by the writer, indicate that the failure envelopes for Vienna clay are fairly close to those for Weald clay, but they do not pass through the point of origin.

Influence of the molding water content. The unified diagram for Vienna clay, Fig. 38-C, does not represent the contours corresponding to low pressures or water contents approaching the molding water content, which resemble those obtained by Henkel for overconsolidated Weald clay. Rendulic also found that a slight swelling occurred at and after failure in drained tests on normally consolidated Vienna clay and, as indicated above, the corrected failure envelopes do not pass through the point of origin. Data available to the writer do not show any case in which a clay normally consolidated from a water content close to the liquid limit exhibits a volume increase at and after failure. The anomalous behavior of a normally consolidated Vienna clay is

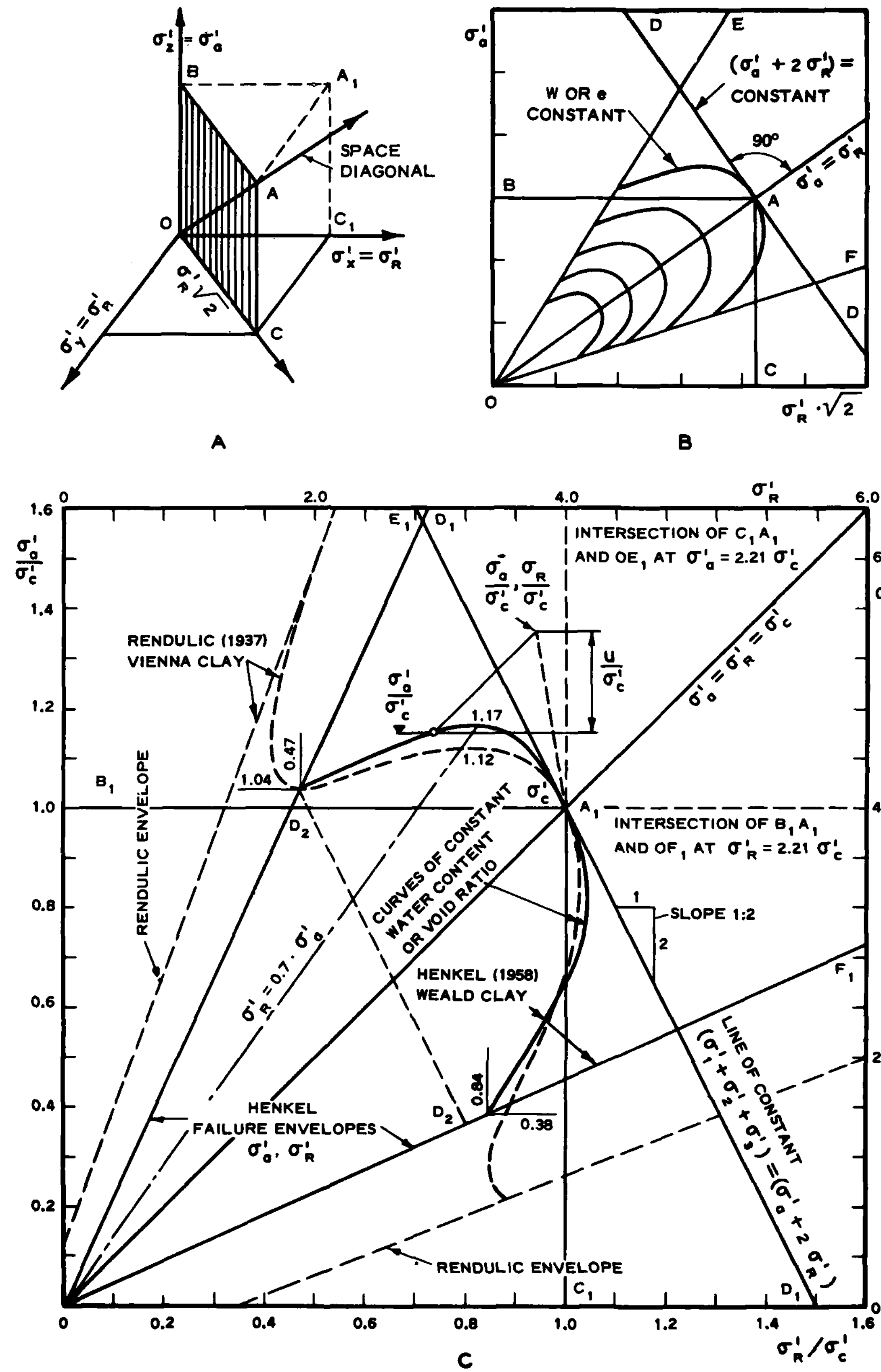

FIG. 38. UNIFIED RENDULIC DIAGRAM – NORMAL CONSOLIDATION

probably caused by the low molding water content used by Rendulic. The slight difference in unified water content contours of Vienna clay and Weald clay before failure may indicate actual differences in soil characteristics, but it may also in part be the result of differences in molding water content or in the locations where the pore-water pressures were measured.

It is often convenient to use a low molding water content in the preparation of triaxial test specimens of remolded saturated clays, but this practice may cause abnormalities in the test results obtained. The influence of the molding water content on volume change and strength characteristics of remolded saturated clays should be investigated, and proper limits should be established for this water content or the liquidity index with respect to the consolidation pressures used in the tests.

Stresses and Void Ratios

The stress conditions at failure of normally consolidated Weald clay, expressed in terms of the equivalent consolidation pressure σ'_c and obtained from the unified diagram in Fig. 38-C, are summarized in the following table.

Table 5

Stress Conditions at Failure of Weald Clay

Stresses	Compression	Extension	Ratio
σ'_a	$1.04\ \sigma'_c$	$0.38\ \sigma'_c$	
σ'_r	$0.47\ \sigma'_c$	$0.84\ \sigma'_c$	
$p' = \frac{1}{3}(\sigma'_a + 2\sigma'_r)$	$0.66\ \sigma'_c$	$0.69\ \sigma'_c$	1.04
$q = (\sigma'_1 - \sigma'_3)$	$0.57\ \sigma'_c$	$0.46\ \sigma'_c$	0.81
$(\sigma'_1 + \sigma'_3) = (\sigma'_a + \sigma'_r)$	$1.51\ \sigma'_c$	$1.22\ \sigma'_c$	0.81
σ'_1/σ'_3	2.21	2.21	1.00
q/p'	0.87	0.67	0.77
u	$0.53\ \sigma'_c$	$0.62\ \sigma'_c$	1.17

The unified diagrams in Fig. 38-C and the numerical data in Table 5 represent generalizations of several contours or tests, and individual test results may deviate therefrom to a minor extent. However, it is believed that the data are sufficiently reliable for a discussion of qualitative relations and trends. As previously mentioned, Henkel found that the angle of inclination of the Mohr envelope is practically the same for compression and

extension tests; that is, the effective stress strengths are nearly identical for the two test conditions. On the other hand, the data in Table 5 show that the constant volume strength in extension tests is only 81 per cent of that obtained in compression tests. This reduction in strength is primarily caused by the development of greater pore-water pressures in extension tests.

Triaxial consolidation produces virgin consolidation diagrams which, in a semilogarithmic plot, are parallel to those obtained by confined consolidation, Fig. 10, provided the initial water contents are equal. It is often stated that the void ratio primarily is a function of the major principal stress, or that the consolidation diagrams plotted as a function of σ_1' are nearly independent of the principal stress ratio, in which case the constant water content contours should be straight and parallel to the lines AB and AC in Fig. 38. It is seen from this figure and Table 5 that the above-mentioned statement is nearly correct for some stress conditions but is an oversimplification for other situations. The major principal stress at failure in compression tests is $\sigma_1' = \sigma_a' = 1.04\ \sigma_c'$ which is in close agreement with the statement, but the major principal stress at failure in an extension test is $\sigma_1' = \sigma_r' = 0.84\ \sigma_c'$. For consolidation with the ratio $\sigma_r' = 0.7\ \sigma_a'$, which corresponds to confined consolidation of clays, $\sigma_1' = \sigma_a' = 1.17\ \sigma_c'$ for Weald clay and $\sigma_1' = \sigma_a' = 1.12\ \sigma_c'$ for Vienna clay; that is, the major principal stress is 12 to 17 per cent greater than that required for all-round pressure.

The change in void ratio or the equivalent consolidation pressure during a drained compression test, $\sigma_r' = \sigma_c' =$ constant, is obtained by extending the line C_1A_1 to intersection with the failure envelope OE_1, which yields $\sigma_{af}' = 2.21\ \sigma_c'$, but the relation $\sigma_{af}' = 1.04\ \sigma_e'$ also applies, hence

$$\sigma_{af}' = 2.21\ \sigma_c' = 1.04\ \sigma_e' \quad \text{or} \quad \sigma_e' = 2.12\ \sigma_c' \quad \text{and} \quad n_c = 2.12 \qquad (55\text{-A})$$

where σ_e' is the equivalent consolidation pressure at failure and n_c is the overconsolidation ratio. The following relations for drained extension tests are obtained in a similar manner,

$$\sigma_{rf}' = 2.21\ \sigma_c' = 0.84\ \sigma_e' \quad \text{or} \quad \sigma_e' = 2.63\ \sigma_c' \quad \text{and} \quad n_c = 2.63 \qquad (55\text{-B})$$

which shows that the volume change during a drained extension test is appreciably greater than that in a drained compression test on a normally consolidated clay.

The diagrams in Fig. 38 show that the void ratio is not a unique function of the mean effective stress, p' , or the first stress invariant, J_1' , since the contours then should be parallel to the lines DD and D_1D_1. Changes in void ratio are a function of changes in both normal stresses and shearing stresses and may possibly be expressed as a function of changes in the mean stress, p' , and the octahedral shearing stress, τ_{oct} . Development of mathematical expressions for the complete Rendulic constant water content contours, or for the change in void ratio caused by an arbitrary change in triaxial stress conditions, would aid the understanding of the failure criteria for clays and would permit more reliable estimates of the settlement of structures founded on massive clay deposits.

HENKEL (1958, 1959, 1960) presents a series of interesting correlations between water contents and stress conditions at failure. Many of the diagrams are similar to those used by RUTLEDGE (1947) and shown in Figs. 19 and 20; but in other diagrams the relations are shown in arithmetical coordinates and can be used for predicting pore-water pressures at failure in undrained tests from the results of drained tests, or void ratio changes in drained tests from the results of undrained tests.

Henkel correlates the water content with the effective mean stress, p' . It was mentioned in the foregoing paragraph that the void ratio is not a unique function of p' ; however, at failure there is a definite relation between σ_1' , σ_3' , and p' , and correlations between w or e and p' at failure can then be established. Although the relations between σ_1' , σ_3' , q , and p' at failure are quite different for compression tests and extension tests, there is but little difference between correlations of w and p' for the two types of tests, because the value of p' at failure in extension tests is only slightly larger than that obtained in compression tests; see Table 5 and line D_2D_2 in Fig. 38-C. However, it should be noted that whereas the value of p' at the start of an undrained test is equal to σ_c' , at failure p' is only 0.66 σ_c' in compression tests and 0.69 σ_c' in extension tests. When the effective mean stress, p' , is kept constant during a test, the test specimen is subjected to pure shear, but a normally consolidated clay undergoes a volume decrease as seen by inspection of the families of constant water content contours for Vienna clay, Weald clay, and London clay. This result is contrary to that obtained by the conventional theory of elasticity, but it has been corroborated by compression and extension tests in which p' actually was kept constant during the tests; see Fig. 18 in HENKEL (1958). Strongly

overconsolidated clays undergo a volume increase during similar tests.

Stresses and Pore-Water Pressures

The pore-water pressure, u, created by an increase in external pressure or total stresses during undrained tests on a normally consolidated clay can be determined by plotting the point $(\sigma_a/\sigma_c' , \sigma_r/\sigma_c')$ in the Rendulic diagram, Fig. 38-C, and drawing a line through this point parallel to OA until it intersects the unified contour. The horizontal or vertical distance between the two points is equal to u/σ_c'. The pore-water pressures at failure in compression and extension tests are shown in Table 5. A similar graphical determination of the pore-water pressure can also be used for overconsolidated clays provided the contour for the particular water content and state of overconsolidation is available. As previously mentioned, other diagrams proposed by HENKEL (1958, 1959) can also be used for prediction of pore-water pressures.

SKEMPTON (1954) proposed that the change in pore-water pressures at failure, Δu, can be determined by the coefficients A_f and B_f and the equation

$$\Delta u = B_f[\Delta\sigma_3 + A_f(\Delta\sigma_1 - \Delta\sigma_3)] \qquad (56\text{-A})$$

The coefficient B_f is close to unity for saturated clays, and the equation is generally used in the simplified form

$$\Delta u = \Delta\sigma_3 + A_f(\Delta\sigma_1 - \Delta\sigma_3) \qquad (56\text{-B})$$

Skempton also suggested that the equation be given a form which indicates the influence of changes in the mean principal stress, p, instead of changes in σ_3.

HENKEL (1958, 1960-B) generalized the above-mentioned equations by introducing stress invariants or octahedral stresses, and suggested the following expression

$$\Delta u = \Delta p + a_f\sqrt{(\Delta\sigma_1 - \Delta\sigma_2)^2 + (\Delta\sigma_2 - \Delta\sigma_3)^2 + (\Delta\sigma_3 - \Delta\sigma_1)^2} \qquad (57\text{-A})$$

which also may be written in the shorter form

$$\Delta u = \Delta p + 3a_f\Delta\tau_{oct} \qquad (57\text{-B})$$

There is considerable difference in values of A_f for compression tests and extension tests, whereas corresponding differences in values of a_f are much smaller and in some cases negligible. Therefore, Eqs. 57 have greater general validity than Eqs. 56, and they represent an advance in available means for predicting pore-water pressures.

The coefficients A_f and a_f vary with the consolidation characteristics and the stress history of the clay. These coefficients decrease with increasing overconsolidation, are zero for the critical degree of overconsolidation, and become negative for strongly overconsolidated clays. The pore-water pressures, u, shown in Table 5, refer to test specimens which have been normally consolidated under all-round pressure and then stressed to failure. Test data for anisotropically consolidated specimens of Weald clay are not available to the writer, but assuming that the unified Rendulic contours also are valid for this case, it is estimated that the values of u are appreciably smaller and those of a_f are considerably larger than the corresponding values for isotropically consolidated test specimens. RENDULIC (1937) found that a stress reversal or a decrease of $(\Delta\sigma_1 - \Delta\sigma_3)$ does not produce a decrease of the pore-water pressure but a small increase of this pressure in case the test specimen is normally consolidated. Likewise, the writer found, HVORSLEV (1937), that a stress reversal during a drained direct shear test on Vienna clay produces a slight additional decrease of the void ratio. These findings are in agreement with Eqs. 57, since the sign of a stress change does not affect the sign of the change in pore-water pressure, but it should be noted that the value of the coefficient a_f is quite small for a stress reversal.

In view of the above-mentioned large variations in the values of the pore pressure coefficients with the stress conditions and stress history of the soil, it is not possible to provide reliable numerical data for practical applications, but the pore pressure coefficients must in each case be determined by tests which simulate the stress history and conditions of the prototype soil as closely as possible. Much more research on the problem of pore-water pressures is needed, and development of more comprehensive mathematical expressions for the triaxial consolidation characteristics of clays, mentioned in the foregoing subsection, will also assist in solving this problem.

The Critical Void Ratio Concept

ROSCOE-SCHOFIELD-WROTH (1958) have advanced a very important concept

concerning the existence of a yield surface and a critical void ratio line for remolded saturated clays which are subjected to loading by means of a constant and low rate of increasing strain. With this type of loading the clay can pass through a yield or failure point without collapse, and continue to deform while the path of stresses and void ratios follow a yield surface until a critical void ratio is reached. Thereafter additional deformations take place without further changes in void ratio, pore-water pressure, and stress conditions, provided the rate of strain is not changed. Very large strains may take place, particularly for overconsolidated test specimens, before the critical void ratio line or ultimate state is attained.

WROTH (1958) states that the peak shear strength or failure condition coincides with the ultimate state on the critical void ratio line for normally consolidated clays, but the peak shear strength observed in drained tests on strongly overconsolidated clays occurs well before the ultimate state is reached. That is, a part of the decrease in void ratio and corresponding increase in strength caused by overconsolidation is active at the moment of failure, and the final increase in void ratio and elimination of the effects of overconsolidation occurs after failure. The peak shear strengths referred to are those actually observed plus a surface energy or dilatation correction, which can be quite large before and still be significant at the peak shear strength, but this correction is zero for the ultimate state where changes in void ratio or pore-water pressure cease.

The authors express the results of triaxial tests by means of the variables p' , q , and e ; the latter may be replaced with the water content, w , for saturated clays. With these triaxial coordinates the yield surface for overconsolidated clays is represented by the equation

$$q = \bar{\mu}_e p' + \bar{c}_z \exp(-\bar{B}e) \tag{58}$$

which is identical in form to Eq. 33, and the coefficients have the same relative meaning but are shown with a bar to indicate that they refer to triaxial consolidation and stress conditions. However, the coefficient $\bar{\mu}_e$ is not a coefficient of effective internal friction because it does not indicate the ratio of shear stresses to normal stress in a failure surface. The member $\bar{\mu}_e p'$ may be called the normal stress component and the member $\bar{c}_z \exp(-\bar{B}e)$ the effective void ratio component. The yield surface terminates at the critical void ratio line, where the variables attain their ultimate

values p_u' , q_u , and e_u . The projection of the critical void ratio line on the (e , p) plane is represented by the equation

$$p_u' = p_h' \exp[\overline{B}(e_h - e_u)] \tag{59-A}$$

and the projection on the (p , q) plane by

$$q_u = \overline{\mu}_s \, p_u' \tag{59-B}$$

where (e_h , p_h') is a known point on the critical void ratio line. Some of the symbols in these equations are different from those used by Roscoe-Schofield-Wroth. The change was made to obtain agreement or avoid conflict with the symbols used in this paper. When the projection on the (e , p) plane is plotted in semilogarithmic coordinates, the critical void ratio line appears as a straight line which is parallel to the virgin consolidation diagram obtained by all-round triaxial pressure, Fig. 39-A. These lines correspond to AA and BB in Figs. 19 and 20, but the critical void ratio line is common to normally consolidated and overconsolidated test specimens. Eqs. 59-A and 59-B correspond to Eqs. 21 and 24 for states of normal consolidation.

Published data on triaxial tests on clays, performed with controlled-strain type of loading, often do not conform to the critical void ratio concept, but Roscoe-Schofield-Wroth point out that the tests in such cases were terminated before the ultimate state was attained, and PARRY (1958) demonstrated that the changes in void ratio or pore-water pressure, at the time the tests were terminated, were in direction of the critical void ratio line. This line has not yet been reached in compression tests on strongly overconsolidated clays, but to do so requires very large strains with consequent uncertainty in proper evaluation of triaxial test results. Furthermore, strongly overconsolidated clays undergo swelling during drained tests, and Roscoe-Schofield-Wroth suggest that the volume changes in this case may tend to become concentrated in a relatively narrow zone, and that the void ratio in this zone is greater than the average void ratio of the entire test specimen. This possibility constitutes another probable source of error in triaxial test, which should be investigated.

The data obtained by the writer in tests on Vienna clay and with controlled stress type of loading, Figs. 19 and 20, diverge appreciably from the critical void ratio concept, especially for strongly overconsolidated test

specimens. The probable cause of this divergence is that the rate of strain increases with increasing stress and creates increasing thixotropic reduction in strength so that complete failure occurs before the ultimate void ratios are attained. It was observed that the void ratio of normally consolidated test specimens decreased after failure, even with test durations of several weeks, and that the void ratio of strongly overconsolidated test specimens increased after failure. Reference is made to Fig. 36, which shows that the void ratio decreased from 25 per cent at to 24 per cent after failure with a corresponding increase in strength after cessation of deformations and the thixotropic disturbance.

The concept of the existence of a critical void ratio line produces a great simplification in the determination of ultimate void ratios and strengths in cases where the rate of strain is limited by natural conditions until the ultimate state is attained, and such cases undoubtedly occur in the field. However, there are other cases where actual failures have a catastrophic character, and where the void ratio and stress conditions at and after failure may be more nearly simulated by those obtained in tests with controlled stress type of loading. To the writer's knowledge, published test data do not permit a reliable comparison of failure and ultimate stresses, strains, and void ratios obtained by very slow tests performed with controlled stress and controlled strain types of loading, and this problem merits careful investigation.

Roscoe-Schofield-Wroth expressly limit their investigations and concepts to triaxial compression tests, $\sigma_1' > \sigma_2' = \sigma_3'$. It is hoped that these investigations will be extended to extension tests, $\sigma_1' = \sigma_2' > \sigma_3'$, and to tests with intermediate values of σ_2' . If this is done, it will undoubtedly be found that the concept of a critical void ratio line is valid for all stress conditions. However, it is probable that some of the coefficients or parameters in Eqs. 58 and 59 will vary with the relative value of σ_2' , since this stress has the same influence on the variables p' and p_u' as σ_1' and σ_3' but appears to have relatively little influence on q and q_u , as discussed in greater detail in the following two subsections.

Triaxial Failure Conditions

The basic Coulomb failure condition, Eq. 3, may be combined with the Mohr failure condition and expressed in terms of the principal stresses by the various forms of Eqs. 9, known as the Mohr-Coulomb failure criterion. The

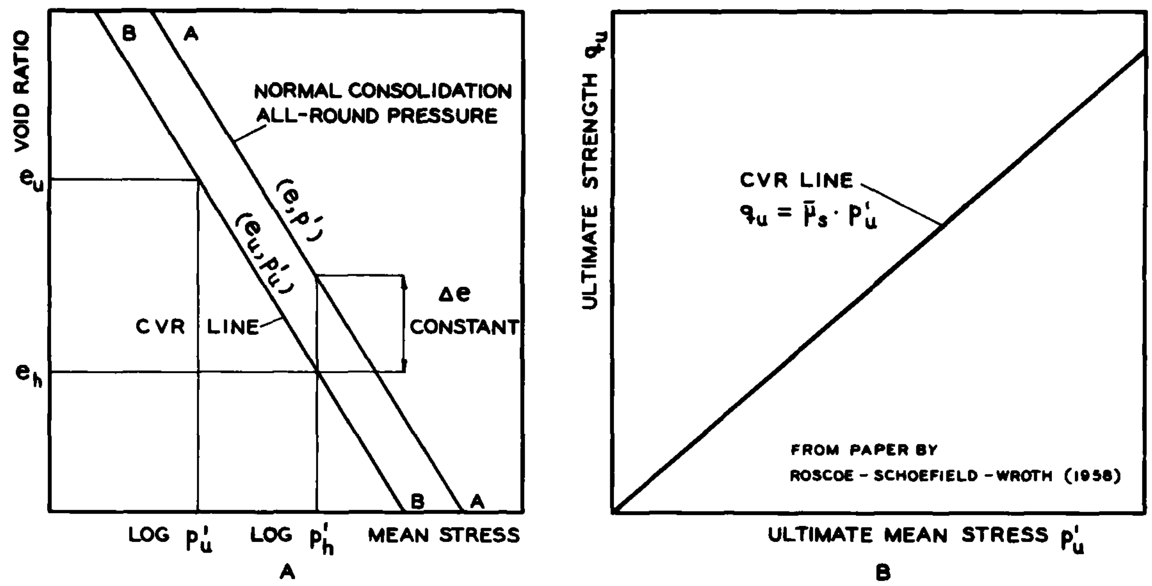

FIG. 39 - THE CRITICAL VOID RATIO LINE FOR COMPRESSION TESTS

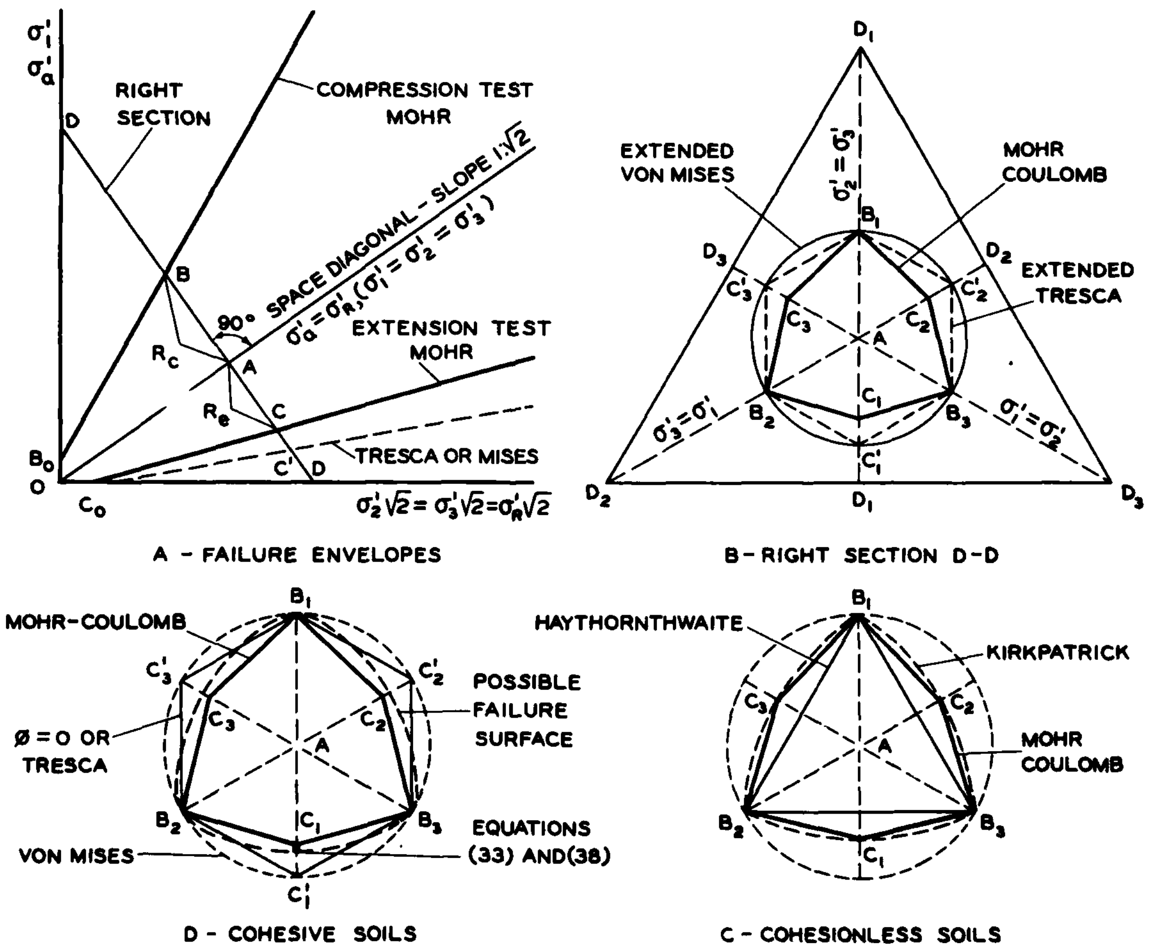

FIG. 40 - FAILURE SURFACES IN THE PRINCIPAL STRESS SPACE

amended Coulomb failure condition with the void ratio as an additional variable, Eqs. 29 and 33, is expressed in terms of principal stresses by Eq. 38 in accordance with investigations by Skempton, Bishop, and Henkel. Finally, Roscoe, Schofield, and Wroth have proposed Eqs. 59 as the ultimate failure condition at large deformations and a constant, slow rate of strain. These equations are not mathematically complete, and more comprehensive expressions for the failure criteria and their representation by lines and surfaces in the principal stress space are discussed in this subsection. The general objective is to examine the limits of validity of the criteria, or to obtain expressions which are valid for all stress conditions without change in form of the equations and the numerical value of the coefficients or parameters.

Failure surfaces. Comprehensive triaxial failure or yield criteria were first developed for metals, crystalline rocks, concrete, and other materials and are described in various textbooks on strength of materials and theories of plasticity; for example, NADAI (1950), PRAGER and HODGE (1951), and TIMOSHENKO (1956). Application of these theories to soils and determination of the corresponding failure surfaces in the principal stress space are described in the excellent papers by RENDULIC (1938) and KIRKPATRICK (1957). These theories and their graphical representation are summarized in the following paragraphs in order to provide a basis for the subsequent discussion.

Fig. 40-A shows a section of symmetry in the principal stress space, similar to that in Figs. 38-A and 38-B. The axes and principal stresses can in turn become the major, intermediate, and minor principal axes and stresses. It is assumed that the failure lines O_cB and O_eC are straight and represent the Mohr-Coulomb failure condition, Eqs. 9, and that the parameters ϕ' and c' are constant or independent of the value of the intermediate principal stress. The right section DD is shown in Fig. 40-B, where D_1, D_2, and D_3 are the points of penetration of the three axes; B_1 represents failure in compression tests with $\sigma_1' > \sigma_2' = \sigma_3'$, and C_1 represents failure in extension tests with $\sigma_2' = \sigma_3' > \sigma_1'$. It may be noted that the change from B_1 to C_1 involves a change in direction of the major principal stress. The points B_2, C_2 and B_3, C_3 are obtained in a similar manner; for example, the point B_3 represents failure for the condition $\sigma_1' = \sigma_2' > \sigma_3'$. It can be shown mathematically that the straight lines connecting these points represent failure conditions for intermediate values of the second principal stress. The failure surface corresponding to the Mohr-Coulomb condition of failure is then a pyramid with the irregular hexagon $B_1C_2B_3C_1B_2C_3$ as base or right

section. This surface is represented by six equations similar to Eq. 9-A or variations thereof, which can be combined into the following complete equation

$$\left[(\sigma_1' - \sigma_2')^2 - (2c'\cos\phi' + (\sigma_1' + \sigma_2')\sin\phi')^2\right]\left[(\sigma_2' - \sigma_3')^2 - (2c'\cos\phi' + (\sigma_2' + \sigma_3')\sin\phi')^2\right]\left[(\sigma_3' - \sigma_1')^2 - (2c'\cos\phi' + (\sigma_3' + \sigma_1')\sin\phi')^2\right] = 0 \qquad (60)$$

The subscript, m , formerly used for ϕ' in the Mohr equations, has been omitted for convenience. If the all-round stress at point A is σ_c' , the stresses at point B are

$$\sigma_a' = \sigma_c' + R_c\sqrt{2/3}\ , \quad \sigma_r' = \sigma_c' - R_c\sqrt{1/6}\ , \quad \text{and} \quad R_c = (\sigma_a' - \sigma_r')\sqrt{2/3}$$

and at point C

$$\sigma_a' = \sigma_c' - R_e\sqrt{2/3}\ , \quad \sigma_r' = \sigma_c' + R_e\sqrt{1/6}\ , \quad \text{and} \quad R_e = (\sigma_r' - \sigma_a')\sqrt{2/3}$$

which inserted in Eq. 9-A yield the following relation between the radii R_c = AB for compression tests and R_e = AC for extension tests

$$\frac{R_e}{R_c} = \frac{1 - \frac{1}{3}\sin\phi'}{1 + \frac{1}{3}\sin\phi'} \qquad (61)$$

It should be noted that this ratio is independent of c' , when this parameter is a constant, and that it approaches unity with decreasing values of ϕ' as shown in Table 6.

Table 6

Ratios of R_c and R_e for Mohr-Coulomb Failure Surface

ϕ'	0°	5°	10°	15°	20°	25°	30°	35°	40°
R_e/R_c	1.000	0.943	0.889	0.842	0.796	0.753	0.715	0.678	0.647
R_c/R_e	1.000	1.061	1.123	1.188	1.258	1.328	1.400	1.473	1.546

For $\phi' = 0$ the Mohr-Coulomb failure criterion reduces to

$$\sigma_1' - \sigma_3' = c' \tag{62}$$

and five corresponding equations which can be combined in a general single equation similar to Eq. 60. This is the Tresca or Guest failure condition, and the corresponding failure surface is a prism, for which the right section is a regular hexagon and $R_e = R_c$. The following failure condition

$$(\sigma_1' - \sigma_3') = c' + \bar{\mu}\,\frac{1}{3}(\sigma_1' + \sigma_2' + \sigma_3') = c' + \bar{\mu}p' \tag{63}$$

and the five corresponding equations have a failure surface which is a pyramid with a regular hexagonal right section. Eq. 63 is called the extended Tresca failure condition, but it is credited to Sandels by JOHANSEN (1958, p. 32).

Theoretical and experimental investigations by von Mises, Hencky, Huber, Lode, Nadai, Schleicher, and others lead to the following failure condition

$$(\sigma_1' - \sigma_2')^2 + (\sigma_2' - \sigma_3')^2 + (\sigma_3' - \sigma_1')^2 = \text{constant} \tag{64-A}$$

which generally is called the von Mises yield or failure condition and applies to many ductile metals or plastic materials. The failure condition is also given in the form

$$(\sigma_1' - \sigma_2')^2 - (\sigma_2' - \sigma_3')^2 - (\sigma_3' - \sigma_1')^2 = [c' + \frac{\bar{\mu}}{3}(\sigma_1' + \sigma_2' + \sigma_3')]^2 \tag{64-B}$$

known as the extended von Mises condition. The members of these equations can easily be replaced with stress invariants or octahedral stresses, and the failure condition may then be expressed in the more general form

$$\tau_{oct} = f(\sigma_{oct}') \tag{64-C}$$

which generally is credited to Schleicher. These equations are symmetrical with respect to the principal stresses; the corresponding failure surfaces are surfaces of revolution with the space diagonal as axis, and the right section is a circle which circumscribes the Tresca hexagon. Some of the above-mentioned equations were originally developed as yield criteria and later also used as failure conditions.

<u>Cohesionless materials.</u> Comprehensive investigations of triaxial failure conditions for soils have primarily been made on sands, and it is of interest to review the results obtained. KJELLMAN (1936) performed tests on sand by means of equipment which permitted independent control of the three principal stresses on a cubical test specimen. The results of compression tests, $\sigma_1' > \sigma_2' = \sigma_3'$, corresponded to $\phi' = 35^\circ$ whereas the results of tests for intermediate values of the second principal stress, $\sigma_1' > \sigma_2' > \sigma_3'$, corresponded to $\phi' = 43^\circ$. It may be noted that Kjellman did not perform extension tests, $\sigma_1' = \sigma_2' > \sigma_3'$.

BISHOP and ELDIN (1953) found good agreement between values of ϕ' obtained in drained compression and extension tests on sand. However, BISHOP (1954) calls attention to the possibility that ϕ' may be slightly larger for intermediate values of the second principal stress, and in support thereof he refers to theoretical investigations of the problem by CAQUOT (1934) and to the results of preliminary investigations at the University of London. Further experiments with the recently developed apparatus for conducting strength tests under conditions of plane strain may yield data which will clarify the problem.

KIRKPATRICK (1957) developed equipment for triaxial tests on thick-walled cylinders of sand, which permits control of the second principal stress. He found close agreement between values of ϕ' for compression and extension tests, but obtained slightly larger values of ϕ' for intermediate stress conditions. The corresponding right section of the failure surface circumscribes the irregular hexagon of the Mohr-Coulomb surface and is shown by the dashed lines in Fig. 40-C. These results are in part corroborated by the above-mentioned limited test data by Kjellman. In contrast thereto, HABIB (1953), PELTIER (1957), and HAYTHORNTHWAITE (1960) found values of ϕ' in extension tests which are considerably smaller than those obtained in compression tests. Haythornthwaite concludes that the Mohr-Coulomb failure condition in some cases may furnish unsafe values and suggests that it be replaced with a criterion based on the triangular surface $B_1B_2B_3$ in Fig. 40-C. These very substantial differences in results obtained by various investigators emphasize the difficulties encountered in performing triaxial extension tests and the need of further investigations of sources of error in triaxial testing equipment and procedures.

<u>Cohesive materials.</u> Compression tests on marble by Kármán and extension tests on the same material by Böker, as reported by RENDULIC (1938) and

JOHANSEN (1959), show that the effective stress strengths in extension tests are 7 to 9 per cent greater than those obtained in compression tests. JOHANSEN (1959) also presents an excellent review of failure conditions for concrete and arrives at the conclusion that these conditions in general agree with the Mohr hypothesis and that there is so much scatter in the test data that it cannot be stated with certainty whether the intermediate principal stress has an appreciable influence on the strength.

RENDULIC (1938) reviews the major yield and failure hypotheses and corresponding surfaces in the principal stress space, but states that his compression and extension tests on Vienna clay do not furnish reliable data on the failure conditions, since the changes in cross section of the test specimens were not determined. On the other hand, he estimates the yield conditions on basis of the shape of the stress paths or contours and suggests that they are represented by a hyperbolic surface of revolution or a variant of the von Mises yield condition, Eqs. 64.

TAYLOR (1948) reiterates a general belief that intermediate principal stress has a minor influence on the strength, and that values of ϕ' obtained in extension tests are about 10 per cent greater than those for compression tests. TAYLOR and CLOUGH (1951) summarize the results of compression and extension tests on undisturbed samples of Cambridge clay. It was found that the undrained or constant volume strength in extension tests is as much as 20 per cent smaller than that obtained in comparable compression tests. This difference in strengths is correctly attributed to differences in pore-water pressures at failure, see Table 5, and it is also suggested that these pressures are a function of the change in the mean principal stress. However, the shear strength lines for critical planes, Fig. 14 in the report, show that the effective stress strengths for extension tests are equal to or slightly larger than those obtained in comparable compression tests. In a later review of these test results, TAYLOR (1955), it is stated that the batch of clay used for the extension tests had physical characteristics which under comparable testing conditions gave about 10 per cent lower strengths than the batch of clay used for the compression tests. Therefore, had all the tests been performed on clay with identical strength characteristics, the effective strengths for extension tests would probably be perceptibly greater than those obtained in compression tests.

HIRSCHFELD (1958) performed undrained triaxial compression and extension tests on undisturbed samples of three different clays. The results obtained

for normally consolidated test specimens are expressed by the effective principal stress ratio at failure, σ_1'/σ_3' , and show considerable scatter because of variations in the physical properties of the individual test specimens. Values of σ_1'/σ_3' for extension tests were in some cases greater and in other cases smaller than those for compression tests, and definite conclusions in regard to the influence of the intermediate principal stress could not be formulated.

The extensive series of triaxial tests on remolded Weald clay, HENKEL (1958, 1959, 1960-A, 1960-B), include both drained and undrained compression and extension tests of various types. The effective stress envelopes for normally consolidated test specimens were practically identical for all types of tests. For overconsolidated test specimens the value of ϕ' for extension tests is about 0.5° smaller than that for compression tests; however, the small cohesion intercept, c' , for extension tests is about 50 per cent greater than that for compression tests.

Referring to Fig. 38-C, Table 5, and Eqs. 55, the relative values of σ_e' or the overconsolidation ratios at failure in extension tests are greater than those in compression tests. According to Eqs. 33 and 38 the effective shear strength lines for extension tests should then lie slightly above those for compression tests. This problem is discussed in greater detail in the following subsection.

Definite conclusions cannot be formulated on the basis of the test data summarized above, but it seems probable that the Mohr-Coulomb hexagon represents the inner limit of the failure surface for clays, Fig. 40-D. Some of the test data and consideration of the void ratios at failure indicate that points representing extension tests may lay slightly outside the Mohr-Coulomb hexagon. Strength data on clay for intermediate values of the second principal stress are not yet available, but considering the results obtained by KIRKPATRICK (1957) for sand, Fig. 40-C, it seems possible that the actual failure surface for clays may have a curved form as shown by the dash-line diagram in Fig. 40-D. With decreasing values of ϕ' , the Mohr-Coulomb hexagon approaches the regular Tresca hexagon, and it is possible that the strength of plastic clays with very small values of ϕ' may be represented by the von Mises circle and extended failure conditions, Eqs. 64.

Comments on the Variables

Stresses on principal or critical planes. The results of triaxial tests

are obtained in terms of principal stresses, and the required number of assumptions is generally reduced when these stresses are used in the evaluation and application of the test results. However, in many investigations and practical applications it is desirable or necessary to express the test results in terms of stresses on theoretical or assumed critical planes or planes of failure. The angle of inclination of such planes, α , is generally determined by Eq. 8, but the assumed values of ϕ' vary. As previously mentioned, minor variations in the values of α have very little influence on the position of the computed shear strength line, but such variations may cause appreciable differences in the individual values of σ_f' and α_f and in correlations between these stresses and consolidation pressures, void ratios, or water contents.

Stress invariants. The use of stress invariants or the therefrom-derived octahedral stresses often appreciably simplifies the expression of relations between stresses, strains, and volume changes, and also simplifies certain yield and failure criteria, such as those by Tresca and von Mises, Eqs. 63 and 64. The complete Mohr-Coulomb failure condition, Eq. 60, which is symmetrical with respect to σ_1' , σ_2' , and σ_3' , can also be expressed in terms of the three stress invariants, but the resultant equation becomes very complicated, as emphasized by JOHANSEN (1958). However, COLEMAN (1960) has recently suggested a relatively simple expression in terms of stress invariants for a curved failure surface passing through the six points of the Mohr-Coulomb hexagon, Fig. 40. The curved and continuously convex failure surface shown in Fig. 40-D may possibly be represented by an equation similar to the one proposed by Coleman. Stress invariants or octahedral stresses should not be used in the partial Mohr-Coulomb failure condition, Eqs. 9, which do not contain the intermediate principal stress.

The consequences of using the first stress invariant, J_1' , or the mean stress, p' , in expressing the results of individual series of strength tests on soils which conform to the Mohr-Coulomb failure criterion may be demonstrated as follows. The intermediate principal stress may be expressed by

$$\sigma_2' = \frac{1}{2}(\sigma_1' + \sigma_3') + \frac{n}{2}(\sigma_1' - \sigma_3') \tag{65-A}$$

where n is a coefficient which varies between -1 and +1. The mean principal stress is then

$$p' = \frac{1}{2}(\sigma_1' + \sigma_3') + \frac{n}{6}(\sigma_1' - \sigma_3') \tag{65-B}$$

and

$$(\sigma_1' + \sigma_2') = 2p' - \frac{n}{3}(\sigma_1' - \sigma_3') \tag{65-C}$$

which introduced in the partial Mohr-Coulomb failure condition, Eq. 9-A, yields

$$(\sigma_1' - \sigma_3') = c' \frac{2 \cos \phi'}{1 + \frac{n}{3} \sin \phi'} + p' \frac{2 \sin \phi'}{1 + \frac{n}{3} \sin \phi'} \tag{66-A}$$

In this equation the coefficients for c' and p' are not constant but vary with the value of n or the intermediate principal stress. For compression tests, or $\sigma_2' = \sigma_3'$ and $n = -1$, Eq. 66-A becomes

$$(\sigma_1' - \sigma_3')_c = c' \frac{2 \cos \phi'}{1 - \frac{1}{3} \sin \phi'} + p' \frac{2 \sin \phi'}{1 - \frac{1}{3} \sin \phi'} \tag{66-B}$$

and for extension tests, or $\sigma_2' = \sigma_1'$ and $n = +1$,

$$(\sigma_1' - \sigma_3')_e = c' \frac{2 \cos \phi'}{1 + \frac{1}{3} \sin \phi'} + p' \frac{2 \sin \phi'}{1 + \frac{1}{3} \sin \phi'} \tag{66-C}$$

The strengths obtained by these equations for equal values of c' and p' have the following ratio

$$\frac{(\sigma_1' - \sigma_3')_c}{(\sigma_1' - \sigma_s')_e} = \frac{1 + \frac{1}{3} \sin \phi'}{1 - \frac{1}{3} \sin \phi'} \tag{67}$$

which is identical with Eq. 61, as it should be since Eq. 66-A has the same form as the extended Tresca failure criterion, Eq. 64-B, for which $R_c = R_e$, whereas $R_c > R_e$ for the Mohr-Coulomb criterion. If the coefficients for c' and p' in Eq. 66-A are determined by compression tests and used without change for estimation of the strength in extension tests, the result will be considerably greater than the actual strength; for example, for $\phi' = 25^\circ$ the overestimation amounts to nearly 33 per cent; see Table 6. Likewise, plotting strengths or principal stress ratios at failure versus corresponding values of J_1' or p' instead of $(\sigma_1' + \sigma_3')$ or σ' may cause a considerable increase in the spacing of the diagrams representing compression and extension tests, with consequent difficulty in utilizing the correlations for intermediate or unknown values of σ_2' .

Weighted intermediate stress. In cases where the actual failure conditions are between those represented by the Mohr and von Mises failure

criteria, unified expressions or diagrams cannot be obtained by use of principal or octahedral stresses or stress invariants. JOHANSEN (1958) proposes, in such cases, that the stresses in a plane for which the direction cosines of the normal are $(1\ ,\ \alpha\ ,\ 1)/\sqrt{2+\alpha^2}$ be used as variables. These stresses are

$$\sigma'_\alpha = \frac{\sigma'_1 + \alpha^2\sigma'_2 + \sigma'_3}{2+\alpha^2} \tag{68-A}$$

and

$$\tau_\alpha = \frac{\sqrt{\alpha^2(\sigma'_1 - \sigma'_2)^2 + (\sigma'_1 - \sigma'_3)^2 + \alpha^2(\sigma'_2 - \sigma'_3)^2}}{2+\alpha^2} \tag{68-B}$$

and the proposed failure condition is then

$$F(\sigma'_\alpha\ ,\ \tau_\alpha) = K \tag{69-A}$$

which may be simplified by observing that, for small values of α , τ_α is approximately equal to $\tau_m = \frac{1}{2}(\sigma'_1 - \sigma'_3)$ or

$$F(\sigma_\alpha\ ,\ \tau_m) = K \tag{69-B}$$

Eqs. 69 are similar to the extended Tresca and von Mises criteria but with weighted values of the intermediate principal stress. As previously mentioned, von Mises and Böker found that the Mohr envelope for extension tests on marble lies above the one for compression tests. JOHANSEN (1958) plotted these test data as functions of σ'_α and τ_m , using $\alpha = 0.4$, and found that the two envelopes then merged into a single curve. The method proposed by Johansen may be used to advantage in case further experiments should show that the actual failure surface for clays lies between those corresponding to the Mohr and von Mises failure criteria.

<u>Cohesion as a function of the void ratio.</u> As suggested by Eqs. 29, 33, and 38, the cohesion in the Coulomb failure criterion may be expressed as a function of the void ratio in order to take the stress history of the soil and the curvature of the shear strength lines into consideration. Fig. 38-C shows that the values of ϕ'_s and σ'_1/σ'_3 at failure are nearly identical for compression and extension tests on normally consolidated test specimens of remolded Weald clay. However, in undrained extension tests the normal stresses

and shear stresses at failure are 81 per cent of those for undrained compression tests, see Table 5, but the void ratios and equivalent consolidation pressures are identical for the two test conditions. This means that the void ratio in extension tests is smaller and the equivalent consolidation pressure is larger than in compression tests with the same effective stresses. In the case of drained tests, the effective stresses at failure are nearly identical, but the equivalent consolidation pressures at failure in extension tests are considerably greater than those in compression tests, Eqs. 55. Therefore the cohesion component and the effective stress strength in extension tests should be greater than in compression tests, or the values of the strength parameters, ϕ'_e and κ, must be different for compression tests and extension tests. The following values of the strength parameters for Weald clay were obtained by PARRY (1956) and are quoted by HENKEL (1958).

Table 7

Effective Friction and Cohesion Parameters for Weald Clay

Type of Test	Not Corrected for Surface Energy ϕ'_e	Not Corrected for Surface Energy κ	Corrected for Surface Energy ϕ'_e	Corrected for Surface Energy κ
Compression tests				
Drained	18°	0.05	20°	0.02
Undrained	20°	0.02	20°	0.02
Extension tests				
Drained	17.5°	0.04	19.5°	0.03
Undrained	16°	0.06	16°	0.06

Values of ϕ'_e for extension tests are slightly smaller and those of κ tend to be slightly larger than corresponding values for compression tests. The difference between the results of drained and undrained tests decreases when corrections are made for the surface energy corresponding to the rate of volume changes at failure but, as previously mentioned, it is also possible that this difference in test results in part may be due to remanent pore-water pressures in drained tests and slight errors in measuring the significant pore-water pressures during undrained tests.

The influence of constant values of the parameters ϕ'_e and κ on the effective stress strengths in compression and extension tests may be estimated as follows for normally consolidated Weald clay. It is assumed that $\phi'_e = 18^\circ$, $\kappa = 0.05$, and that the inclination of the failure planes is $\alpha = 45 - \frac{1}{2}\phi'_e$.

The shear strength is then, according to Eq. 33,

$$\tau_f = \sigma'_f \tan \phi'_e + \kappa \, \sigma'_e = 0.325 \, \sigma'_f + 0.05 \, \sigma'_e \tag{70}$$

Assuming $\sigma'_1/\sigma'_3 = 2.21$ for compression tests, Table 5, the normal effective stress at failure is

$$\sigma'_f = \frac{1}{2}(\sigma'_1 + \sigma'_3) - \frac{1}{2}(\sigma'_1 - \sigma'_3) \sin \phi'_e = 1.42 \, \sigma'_3 \tag{71}$$

For drained compression tests $\sigma'_3 = \sigma'_c$, and according to Eq. 55-A

$$\sigma'_e = 2.12 \, \sigma'_c = 2.12 \, \sigma'_3$$

or by use of Eq. 71

$$\sigma'_e = \sigma'_f \, 2.12/1.42 = 1.49 \, \sigma'_f \tag{72}$$

The shear strength in drained compression tests is then

$$\tau_{fc} = 0.325 \, \sigma'_f + 0.005 \times 1.49 \, \sigma'_f = 0.400 \, \sigma'_f \tag{73}$$

It is tentatively assumed that Eq. 71 also is valid for extension tests, and Eq. 55-B then yields

$$\sigma'_e = \sigma'_f \, 2.63/1.42 = 1.85 \, \sigma'_f$$

and

$$\tau_f = 0.325 \, \sigma'_f + 0.05 \times 1.85 \, \sigma'_f = 0.418 \, \sigma'_f \tag{74}$$

This equation corresponds to $\tan \phi'_s = 0.418$ or $\phi'_s = 22^\circ 41'$ and $\sigma'_1/\sigma'_3 = 2.26$. Recomputation with this value of the principal stress ratio yields

$$\sigma'_f = 1.43 \, \sigma'_3 \qquad \text{and} \qquad \sigma'_e = \sigma'_c \, 2.26/0.84 = 2.69 \, \sigma'_c = 1.88 \, \sigma'_f$$

The shear strength in drained extension tests is then

$$\tau_{fe} = 0.325 \, \sigma'_f + 0.05 \times 1.88 \, \sigma'_f = 0.419 \, \sigma'_f \tag{75}$$

The strengths expressed by Eqs. 73 and 75 are comparable effective stress strengths, and their ratio is

$$\tau_{fe}/\tau_{fc} = 0.419/0.400 = 1.048 \qquad (76)$$

That is, the effective stress strength in extension tests is 4.8 per cent higher than that in compression tests. In undrained tests $\sigma_e' = \sigma_c'$ for both compression and extension tests and σ_f' varies, but the ratio τ_{fe}/τ_{fc} is also 1.048, when comparisons are made for the same value of σ_f'. Similar computations for $\phi_e' = 20^\circ$ and $\kappa = 0.02$ yield $\tau_{fe}/\tau_{fc} = 1.02$.

As mentioned in Section 5, much greater values of κ are obtained in direct shear tests than in triaxial tests. Direct shear tests on Vienna clay yielded $\phi_e' = 17^\circ 30'$ and $\kappa = 0.10$. Assuming for the purpose of illustration that these values of the parameters are valid for triaxial tests on a clay with characteristics similar to those of the Weald clay, and taking the shape of the Rendulic curves for Vienna clay into consideration, Fig. 38-C, it is estimated that $\tau_{fe}/\tau_{fc} = 1.09$, which constitutes a perceptible difference in strengths.

It appears beyond reasonable doubt that the void ratio at failure in extension tests is smaller than that in comparable compression tests. According to the hypothesis formulated by Eq. 33, such a difference in void ratios should cause extension tests to yield slightly higher effective stress strengths than compression tests. As shown above, the difference between theoretical values of τ_{fe} and τ_{fc} is very small and of the same order of magnitude as the difference between some of the test results shown in Table 7. Therefore, it is difficult to verify or disprove the correctness of the theoretical difference in strengths by experiments. It is possible, as shown by Parry, that the strength parameters vary slightly with the stress conditions. However, considering the sources of error and difficulties encountered in triaxial testing, it is also possible that future tests with improved equipment and procedures may verify that triaxial extension tests yield effective stress strengths which are slightly higher than those obtained in compression tests.

10. CONCLUSION AND SUMMARY

The principal topic of this paper is the shear strength of saturated, remolded, and reconsolidated clays, but a major part deals with related properties which influence the shear strength. The paper is in part a restatement of the results of earlier investigations by the writer and in part a review of recent research by others. In conformity with the objectives of this conference, emphasis is placed on discussion of fundamental relations rather than on methods for determination and application of shear strength of clays in practical problems.

The Shear Strength

Mohr-Coulomb failure criteria. The shear strength of clays is commonly determined by the Coulomb or Mohr failure criteria. When these criteria are expressed in terms of total stresses, the parameters are subject to such wide variations that they must be determined by tests which closely simulate stresses, stress history, drainage conditions, and time in the prototype structure. Nevertheless, failure criteria expressed in terms of total stresses are often convenient to use in the solution of practical problems, but their limitations should be realized.

The same failure criteria expressed in terms of effective stresses in accordance with the Terzaghi concept, Eqs. 3 and 9, have much greater general validity. Even then the parameters vary with the stress history of the clay, and practical applications require estimation of the pore-water pressure or effective stresses at failure. A graphical representation of the failure criteria consists of a shear-strength line for normally consolidated clays plus a family of hysteresis curves for overconsolidated clays, Fig. 6.

Stress-void ratio failure criteria. In research extending investigations by Terzaghi and Janiczek, the writer found that the strength of remolded and reconsolidated clays can be expressed as an effective friction component, which is a function of the effective normal stress, plus an effective cohesion component, which is a function of the void ratio or the equivalent consolidation pressure at failure, Eqs. 29 and 33. This concept has been extended to triaxial tests and stress conditions by Skempton, Bishop, and Bjerrum, Eqs. 38.

The parameters of the stress-void ratio failure criterion are independent of the stress history of the clay, and a graphical representation of the criterion is reduced to a single straight line, Figs. 24 and 27. This

represents a considerable simplification from a theoretical viewpoint, but application of the criterion to practical problems requires estimation of pore-water pressures and/or void ratios at failure, which in many cases is not convenient or even possible considering current knowledge of the volume-change characteristics of clays. However, the stress-void ratio criterion offers a consistent explanation of the deficiencies of the commonly used and simpler failure criteria, and it facilitates estimation of corrections to and the limits of application of these criteria.

The effective friction and cohesion components may be considered as physical or phenomenological components, which possibly are related to primary, secondary, and remanent changes in spacing of the clay particles, but probably not to separate types of intrinsic forces. The analytical expression for the effective cohesion component, Eq. 37-A, is probably a simplification of the actual relations, and its validity may be limited to the normal ranges of void ratios or water contents of a given clay.

Strength and void ratio. The shear strength of normally consolidated clays, excepting some very fat and active clays, can be expressed as an explicit function of the void ratio at the start of a test or the void ratio at failure, Eqs. 26-27 and Fig. 19, but the parameters in these functions, or the strengths at constant void ratio, vary with the relative value of the intermediate principal stress at failure, Fig. 38 and Table 5. Similar expressions for the strength of overconsolidated clays are much more complicated, or have a limited range of validity, because this strength also is a function of the stress history of the clay.

The above-mentioned simple relations between strength and void ratio are valuable from a theoretical standpoint and when dealing with uniform clays, but they may be difficult to apply to many undisturbed clay deposits in which there are relatively great variations in water content and composition with short distances even though the variations in strength are small.

Inclination of planes of failure. Possible variations in the shear strength parameters or the effective cohesion component with direction have been investigated by measurement of the angles of inclination of failure planes in unconfined and triaxial compression tests, Fig. 28, but definite conclusions cannot be formulated. Anisotropy and minor irregularities may have an appreciable influence on the inclination of these planes, Fig. 29, and there is considerable scatter in measured values. However, the average values of the angles of inclination for remolded clays are in much better agreement

with the stress-void ratio criterion than with the Mohr-Coulomb failure criterion. Published data on angles of inclination of failure planes in undisturbed clays are limited in extent and vary much more than those for remolded clays.

Volume-change energy. It has been proposed that the effective friction and cohesion components be modified by introduction of a surface energy correction or component, Fig. 9, which is a function of the rate of volume change at failure, Eqs. 11-12. However, the origin or dissipation of the volume-change energy and its relation to electrochemical and external forces need clarification. The rate of volume change at failure may indicate the existence of corresponding, remanent excess pore-water pressures in slow drained tests.

Time-dependent decrease in strength. An increase in shear stresses causes a decrease of the pore-water pressures and an initial increase in strength of strongly overconsolidated clays. A subsequent equalization of the pore-water pressures causes an increase in void ratio, a decrease in effective stress, and a decrease in strength. Rapid or undrained tests on strongly overconsolidated clays yield greater strengths than slow drained tests on the same clays, Fig. 31. These phenomena are explained by the volume-change characteristics of clays and the stress-void ratio failure criterion.

A secondary or rheological decrease in strength occurs irrespective of the state of consolidation which, according to investigations by Casagrande, Wilson, and others, is proportional to the logarithm of time, Fig. 33 and Eq. 48. This decrease in strength is in part caused by a secondary increase in pore-water pressures and a corresponding decrease of the effective stresses and the effective friction component. Another part is caused by the existence of a "rheological component" which decreases to zero with increasing time. This component is in part a viscous phenomenon, but may be the result of various factors which have not yet been clarified.

It is assumed in this paper that the rheological component is a part of the effective cohesion component, and that the remaining part of this component constitutes the ultimate cohesion component, Fig. 9. Available experimental data are inadequate for verification of this assumption and for determination of the individual parts of the total decrease in strength. It is possible that the ultimate cohesion component may be negligible for some clays, and may constitute an appreciable part of the effective cohesion component for other clays.

Principal stress space. Comprehensive failure conditions and corresponding failure surfaces in the principal stress space are reviewed in the preceding section of the paper. The relative value of the intermediate principal stress affects the pore-water pressures and the constant volume strength to a considerable degree, Fig. 38 and Table 5, but appears to have only little influence on the failure conditions expressed in terms of principal stresses. The Mohr-Coulomb failure surface probably represents the lower limits of the shear strength, and it is possible that the actual failure surface for clays is a curved surface which circumscribes the Mohr-Coulomb hexagon, Fig. 40. However, definite conclusions cannot yet be formulated because of scatter in some of the limited published data available.

Influence of deformations. Geuze has suggested that shear stresses exceeding a certain "flow limit" cause continuing deformations and ultimate failure of a clay. Goldstein has suggested that failure occurs when the shear strains exceed a certain limiting value. The design of many foundation structures is governed by the deformations rather than by the limiting strength of the soil, which often is taken into consideration by using allowable strengths which correspond to small but arbitrarily selected strains in stress-strain diagrams obtained by standard laboratory tests. These suggestions and methods need to be amplified by further detailed and systematic research utilizing the methods and theories of plasticity and rheology.

Shear strength after failure. A permanent decrease in strength after failure may be caused by an increase in void ratio or a change in structure of the clay. However, a major part of the decrease in strength after failure of remolded clays is caused by a transient increase in pore-water pressure and/or thixotropic changes in strength, in which case the strength is regained in time after the deformations cease. The magnitude of this reversible decrease in strength depends to a large extent on the rate of deformation at and after failure.

Properties Influencing the Shear Strength

Structure of clay. Recent investigations by Lambe, Rosenquist, Tan, and others constitute a significant advance in clarification of the nature of the intrinsic forces acting in the clay-water system and of the many factors which govern the magnitude of these forces. The intensity of the forces decreases rapidly with distance from a clay particle, and the minimum distance between the particles is of greater importance than the average distance. The

resultant intrinsic force between particles of a given clay is a function of both the void ratio of the clay and the structural arrangement of the particles. Initial variations in structure may have considerable influence on the deformation characteristics and the pore-water pressures, but investigations by Seed, Mitchell, and Chan indicate that such variation may have relatively little influence on failure conditions expressed in terms of effective stresses, probably because the deformations tend to produce similar structures at failure.

Changes in void ratio. Volume-change characteristics of clays during drained tests are fairly well known from a qualitative standpoint. An increase of the mean, effective normal stress produces a decrease in void ratio and vice versa. An increase in shear stresses causes a decrease in void ratio of normally consolidated and lightly overconsolidated clays but an increase in void ratio of strongly overconsolidated clays. A shear stress reversal or decrease in shear stresses may produce void ratio changes of the same sign but much smaller magnitude as an increase in shear stresses. The volume decrease caused by a change in shear stresses contradicts the results obtained by the mathematical theory of elasticity, and this fact should be taken into consideration when the theory and related failure criteria are applied to clays.

Roscoe, Schofield, and Wroth have demonstrated that void ratios and effective stress conditions of normally consolidated and overconsolidated clays approach the same "critical void ratio" line at very large strains. This ultimate state may be reached after failure or may coincide with the peak shear strength of normally consolidated clays, depending upon the testing procedure. However, in drained tests on strongly overconsolidated clays the critical void ratio is attained after failure for both controlled strain and controlled stress types of loading.

Quantitative data on volume changes of clays have been obtained by confined consolidation tests and by triaxial compression and extension tests but not yet for stress conditions corresponding to plane strain or intermediate values of the second principal stress. The common statement that the void ratio primarily is a function of the major principal stress is approximately correct for some stress conditions but is an oversimplification for other stress conditions, Fig. 38. The change in void ratio during a drained shear test on a normally consolidated clay is a constant for a given clay and testing procedure, Fig. 21, but this change in void ratio varies with the stress

conditions at the start of the test and with the relative value of the intermediate principal stress at failure, Fig. 38.

Analytical expressions have been developed for changes in void ratio during confined consolidation and hydrostatic consolidation tests. Changes in void ratio for other stress conditions corresponding to those in triaxial compression and extension tests can be determined graphically by means of the Rendulic constant void ratio contours and other methods suggested by Henkel. The change in void ratio of clays is a function of changes in both the effective normal stresses and the shear stresses, and efforts should be made to develop a comprehensive theory of volume changes which will permit estimation of the change in void ratio caused by an arbitrary change of any principal stress.

Changes in pore-water pressure. Changes in void ratio under drained conditions and in pore-water pressure under undrained conditions are interdependent. An increase of the mean normal stress tends to produce an increase in pore-water pressure and vice versa. A change in shear stresses tends to produce an increase in pore-water pressures of normally consolidated and slightly overconsolidated clays and a decrease in the pore-water pressure of strongly overconsolidated clays. Upon completion of the initial or primary change in pore-water pressure, a sustained change in stresses produces an additional or secondary change in pore-water pressure which increases with time, as demonstrated by Bjerrum and associates. This phenomenon is related to the rheological properties of clays, but has not yet been investigated in adequate detail.

Previously mentioned methods and diagrams for graphical determination of changes in void ratio can also be used for estimation of changes in pore-water pressure under undrained conditions, Fig. 38. In recently improved analytical methods by Skempton and Henkel, the change in pore-water pressure at failure is expressed as a linear function of the changes in the mean normal stress and in the octahedral shear stress, Eq. 57, but the coefficients in this equation vary with the stress conditions at the start of the test, with the relative value of the intermediate principal stress at failure, and with the stress history of the clay. Therefore, these coefficients must be determined by tests which simulate the stress conditions and state of consolidation in the prototype structure.

Development of more comprehensive methods and theories for determination of changes in void ratio and pore-water pressures is needed to enable a more

general application of the principle of effective stresses to the solution of problems dealing with deformations, bearing capacity, and slope stability of clays. Such developments will also improve the accuracy of estimates of settlements of foundations on massive clay deposits.

The rheological properties. Plastic deformations or creep of clays over extended periods of time often occur in natural deposits and have been observed and recorded. The results of very limited laboratory investigations indicate that the plastic deformations, depending on the stress conditions and type of clay, may (1) ultimately cease, (2) continue for an indefinite period of time at a decreasing rate, and (3) attain a fairly constant velocity which ultimately may increase and cause failure of the clay, Fig. 30 and Eqs. 42-44. However, available experimental data are inadequate for estimating the type of deformation which may take place under field conditions.

Theories of plasticity and rheology have been developed and verified experimentally for other materials. These theories may also be applied to clays, provided the special volume-change characteristics and thixotropic properties of clays are taken into consideration. Adaptations of these theories and the corresponding rheological models to clays have been proposed by Geuze, Tan, and Schiffman, but additional systematic experiments of long duration are required in order to verify the hypotheses and to separate the influence of many factors which affect the plastic deformations of clays.

Fundamental research into the rheological properties of clays should also yield data on the nature and extent of the time-dependent decrease in strength on the existence and values of yield limits, and for improvement of currently used practical expedients for estimating stress conditions which will not cause excessive ultimate deformations.

Sources of Systematic Variations and Errors

The following testing procedures, definitions, and other factors may cause systematic variations or errors in the test results and are in need of additional investigations.

1. Nonuniform distribution of changes in stresses, strains, void ratios, and pore-water pressures in a triaxial test specimen.
2. Review and critical comparison of various definitions of the state of failure and corresponding strengths and strains.
3. Influence of remolding water content in relation to reconsolidation pressure on stresses, void ratios, and pore-water pressures at failure.

4. Influence of the direction of anisotropy or a preferred orientation of clay particles on strengths, deformations, and pore-water pressures.
5. Remanent pore-water pressures in slow drained tests and their relation to the rate of volume change at failure.
6. Influence of the type of load application in very slow tests--controlled stress increments versus controlled rate of strain--on strengths, strains, void ratios, and pore-water pressures at and after failure.

When comparing the results of various test series and observations in both laboratory and field, the possible existence and influence of compensating factors or errors should be carefully investigated and taken into consideration.

Conclusion

The papers and reports referred to in this review constitute only a part of published data on the shear strength of saturated clays. Much additional and pertinent data can undoubtedly be obtained from the detailed test records which have served as a basis for published summaries of the test results. Nevertheless, full understanding of the strength characteristics of clays, and the application of this knowledge to practical problems require extensive additional research, not only into the shear strength proper, but even more into related properties of clays.

The two general problems, concerning properties which influence the shear strength, in greatest need of additional experimental and/or mathematical investigations are (1) relations between changes in stresses, void ratios, and pore-water pressures, and (2) the rheological properties of clays, including the time-dependent decrease in strength.

The above-mentioned research will require very accurate experiments and relatively large quantities of uniform clay for preparation of test specimens. Therefore, it will undoubtedly be advantageous and even necessary to perform some of the experiments on remolded and reconsolidated clays, but these experiments should be supplemented by tests on undisturbed clays, and the final results should be verified and amended as necessary by comparisons of the estimated and actual behavior of clays under field conditions.

It is possible and even probable that the more comprehensive failure criteria and theories for deformation and changes in void ratio and pore-water pressures of clays may be relatively complicated and difficult to apply to practical problems. A single, easily applicable failure criterion for all

possible field conditions probably does not exist, but it should be possible to develop simple failure criteria for several limited ranges of conditions, or to use current practical methods and criteria with appropriate adjustments and delimitations. The more comprehensive failure criteria and theories should be of great assistance in explaining and correcting the deficiencies in currently used practical methods, in the development of other practical methods, in estimating adjustments required by differences between stress conditions in a test specimen and the prototype structure, and in stipulating the proper limits of application of data obtained by various types of tests and field observations.

NOTATIONS

In establishing the following list of notations, efforts have been made to obtain agreement with the letter symbols suggested by the ASCE-ASTM Committee on Glossary of Terms and Definitions in Soil Mechanics and also with the list of symbols recommended for use in papers for the Fifth International Conference on Soil Mechanics and Foundation Engineering. The prime marks used for σ' , c' , and ϕ' indicate effective stresses or values of c and ϕ obtained for relations expressed in terms of effective stresses. The prime mark has been omitted in cases where it is believed that the omission will cause no misunderstandings.

A	area
A_b	partially disturbed area in box shear tests
A_e	effective area in box shear tests
A_f	pore pressure coefficient, Skempton
a_f	pore pressure coefficient, Henkel
B	Terzaghi compression index (equation in natural logarithms)
$\bar{B}$	Terzaghi compression index for all-round triaxial pressure
B_f	pore pressure coefficient, Skempton
C_c	compression index (equation in common logarithms)
c	cohesion, total stresses
c'	cohesion, effective stresses
c_e	effective cohesion or cohesion component
c_{et}	effective cohesion component at time t
c_{ew}	effective cohesion for water content w
c'_m	intercept of Mohr envelope, effective stresses
c_r	cohesion intercept, total stresses
c'_r	cohesion intercept, effective stresses
c_u	ultimate cohesion component
c_v	coefficient of consolidation
c_v	rheological strength component
c_z	coefficient = theoretical effective cohesion at zero void ratio
$\bar{c}_z$	coefficient of effective void ratio component, triaxial
e	void ratio
e_c	void ratio at end of consolidation or start of shear test
e_f	void ratio at failure
e_o	void ratio corresponding to σ'_o (1 kg/cm^2)

e_r	ultimate void ratio after failure
Δe	change in void ratio during shear test
exp	exponential function
f_f	flow limit of τ
f_o	yield limit of τ
G_s	specific gravity of soil solids
G_w	specific gravity of water
H	thickness of test specimen
H_e	effective thickness of test specimen
J_1	J_2 and J_3 = stress invariants
K_a	coefficient of earth pressure
K_o	coefficient of earth pressure at rest
L	side length of test specimen in box shear tests
L_i	liquidity index
ln	natural logarithm, $\log_e$
log	common logarithm, $\log_{10}$
n	porosity
n	coefficient of relative value of σ_2'
n_c	overconsolidation ratio
n_p	prestress ratio
p	mean total normal stress = $\sigma_{oct} = (\sigma_1 + \sigma_2 + \sigma_3)/3$
p'	mean effective normal stress = $\sigma_{oct}' = (\sigma_1' + \sigma_2' + \sigma_3')/3$
p_u'	ultimate mean effective stress at CVR line
q	compressive strength = $(\sigma_1' - \sigma_3')$, subscripts as noted in text
q_t	compressive strength at time t
q_u	ultimate compressive strength at CVR line
$\ddot{q}$	decrease in compressive strength for log cycle of time
R_c	failure surface radius for compression tests
R_e	failure surface radius for extension tests
T_f	actual duration of shear test
T_s	equivalent duration of test for uniform rate of loading
t	time, subscripts as noted in text
t_o	time corresponding to zero strength
u	pore-water pressure
w	water content
w_c	water content at end of consolidation or start of shear test
w_f	water content at failure

w_o	water content corresponding to σ'_o (1 kg/cm^2)
Δw	change in water content during shear test
x	lateral deformation or displacement during shear test
y	vertical settlement or swelling during shear test
α	angle between plane of failure and major principal stress
α	direction cosine or weighting coefficient for σ'_2
α_e	angle of inclination corresponding to ϕ'_e
α_m	angle of inclination corresponding to ϕ'_m
α_o	optimum angle of inclination of plane of failure
α_s	angle of inclination corresponding to ϕ'_s
β	angle of orientation of clay particles or stratifications
β_3	angle of inclination, $1/2(\sigma'_1 - \sigma'_3)$ versus σ'_3
β_s	angle of inclination, $1/2(\sigma'_1 - \sigma'_3)$ versus $1/2(\sigma'_1 + \sigma'_3)$
γ	shear strain
γ_d	shear strain one day after load application
γ_t	shear strain at time t
$\ddot{\gamma}$	increase in shear strain for log cycle of time
ϵ	compressive or tensile strain
η	coefficient of structural viscosity
θ	twist in radians during torsion shear tests
κ	coefficient of effective cohesion
κ_t	coefficient of void ratio component in triaxial tests
κ_R	coefficient of effective cohesion for residual strength
μ_e	coefficient of effective friction = tan ϕ'_ϵ
μ_r	coefficient of shear strength for overconsolidation = tan ϕ'_r
μ_s	coefficient of shear strength for normal consolidation = tan ϕ'_s
$\bar{\mu}_e$	coefficient of effective stress component in triaxial tests
$\bar{\mu}_s$	coefficient of total strength at CVR line in triaxial tests
ν	Poisson ratio
ρ	coefficient of rheological decrease in strength, subscripts in text
ρ_1	value of ρ for t = 1 minute
ρ_m	value of ρ for t = 1000 minutes
σ	total normal stress; subscripts as for σ'
σ'	effective normal stress
σ'_a	effective axial stress in triaxial tests
σ'_c	effective consolidation pressures
σ'_e	equivalent consolidation pressure

σ'_f effective normal stress on plane of failure at failure
σ'_o effective consolidation pressure corresponding to e_o (1 kg/cm^2)
σ'_p effective prestress or preconsolidation pressure
σ'_r effective radial stress in triaxial tests
σ'_{oct} effective octahedral normal stress = p'
σ'_1 σ'_2 and σ'_3 = effective principal stresses
τ shear stress
τ_a average shear strength in plane of failure
τ_b minimum shear strength in plane of failure
τ_c maximum shear strength in plane of failure
τ_d dilatation or surface energy component of shear strength
τ_f shear strength
τ_{fc} shear strength in compression tests
τ_{fe} shear strength in extension tests
τ_o shear strength corresponding to σ'_o
τ_{oct} octahedral shear stress
τ_R residual shear strength
τ_ϕ friction component of shear strength
ϕ angle of shear strength, total stresses
ϕ' angle of shear strength, effective stresses
ϕ'_e effective angle of internal friction
ϕ'_{eR} effective angle of internal friction for residual strength
ϕ'_m angle of inclination of Mohr envelope
ϕ'_r angle of shear strength for overconsolidation
ϕ'_s angle of shear strength for normal consolidation
ϕ_α equivalent angle of internal friction = $(90 - 2\alpha)$

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